Elliptic flow fluctuations
in 200 GeV Au+Au collisions at RHIC

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for the PHOBOS collaboration

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PHOBOS collaboration

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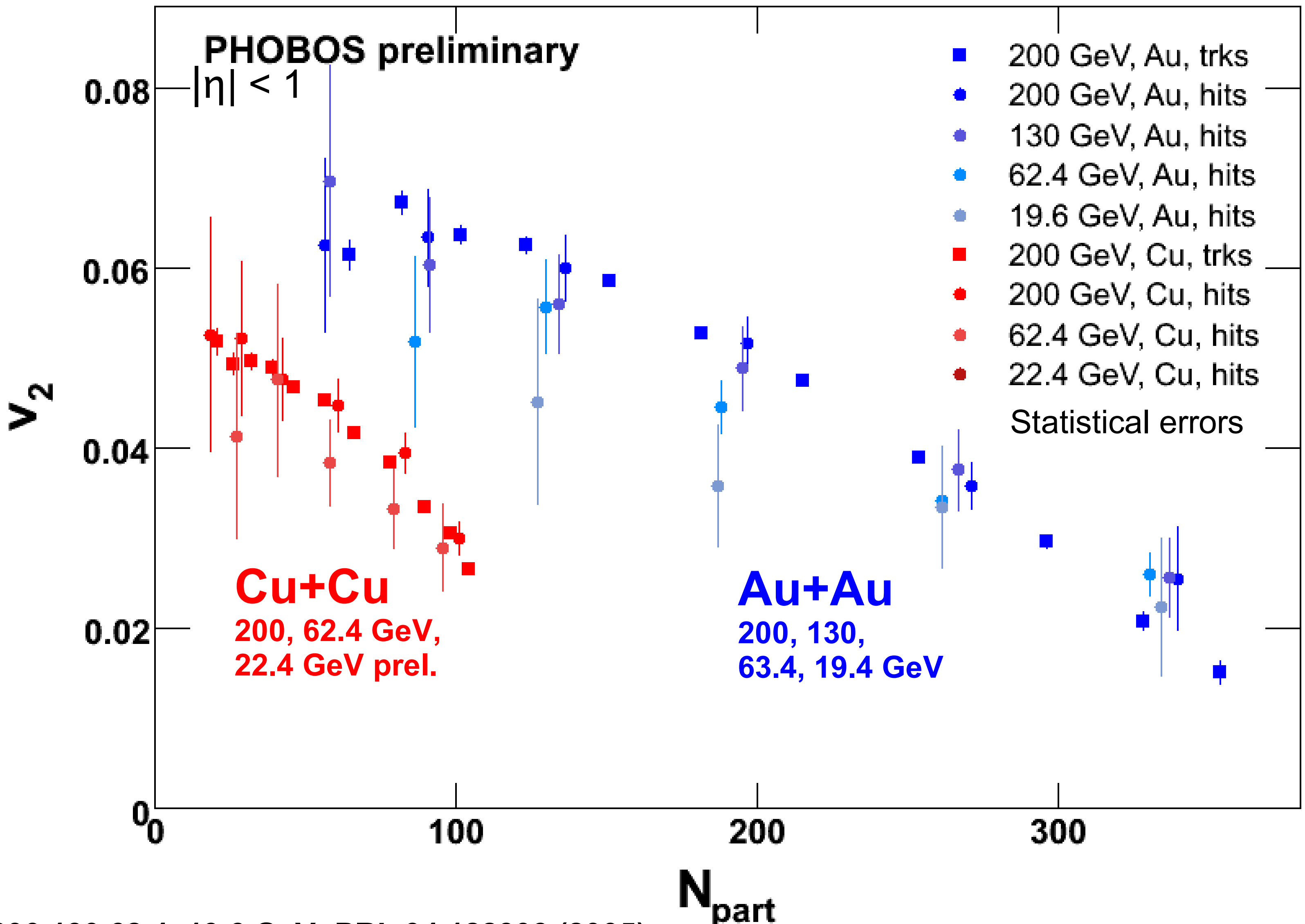
46 scientists, 8 institutions, 9 PhD students

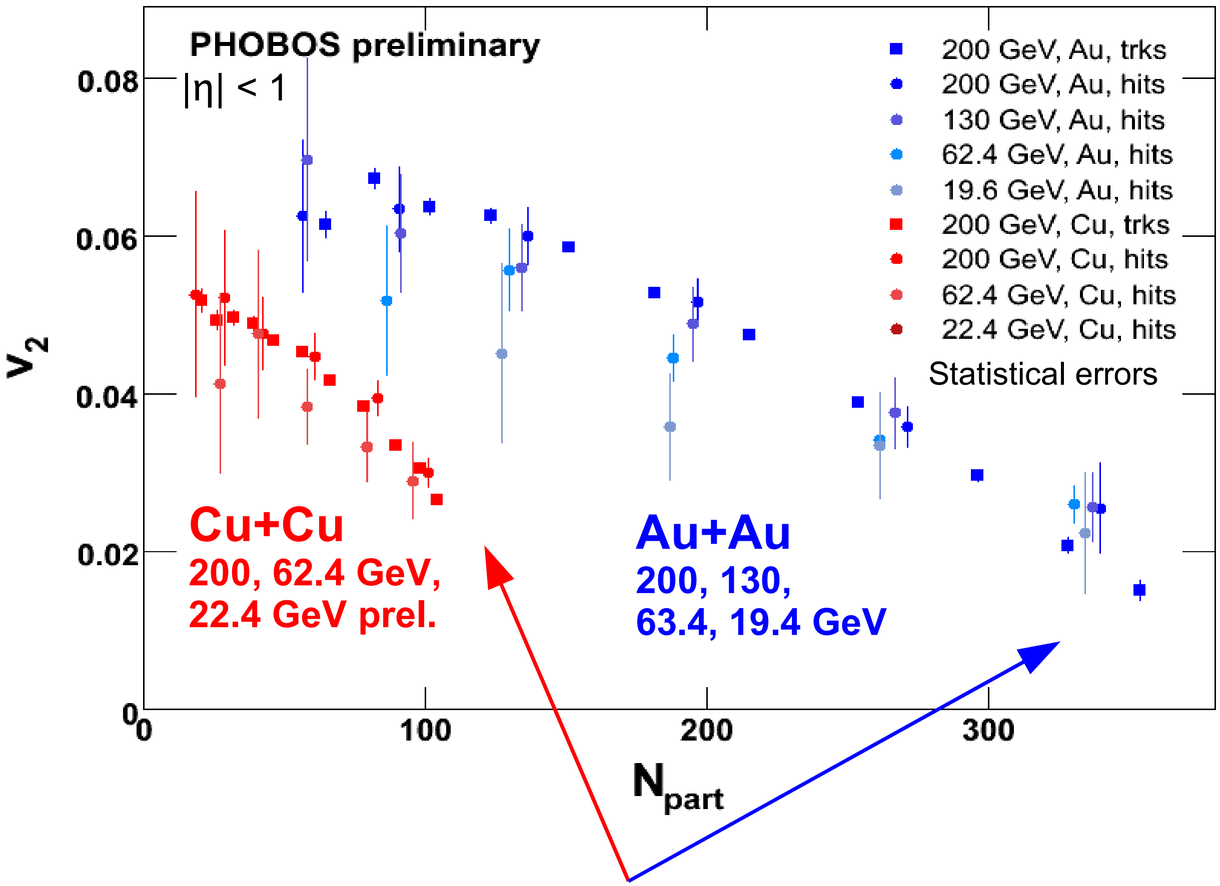
ARGONNE NATIONAL LABORATORY
INSTITUTE OF NUCLEAR PHYSICS PAN, KRAKOW
NATIONAL CENTRAL UNIVERSITY, TAIWAN
UNIVERSITY OF MARYLAND
BROOKHAVEN NATIONAL LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
UNIVERSITY OF ILLINOIS AT CHICAGO
UNIVERSITY OF ROCHESTER

Elliptic flow for different species



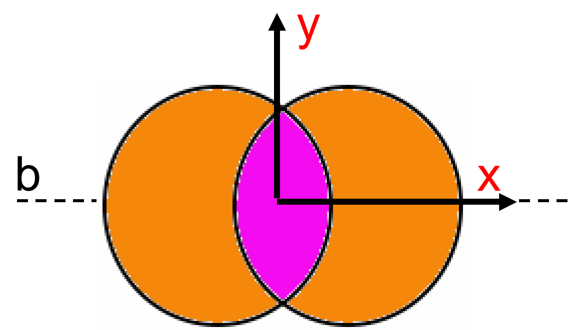
Au+Au, 200,130,62.4+19.6 GeV: PRL 94 122303 (2005)
Cu+Cu, 200+62.4 GeV: nucl-ex/0610037 (sub.to PRL)
Cu+Cu, 22.4 GeV: prel. QM06

Elliptic flow and standard eccentricity

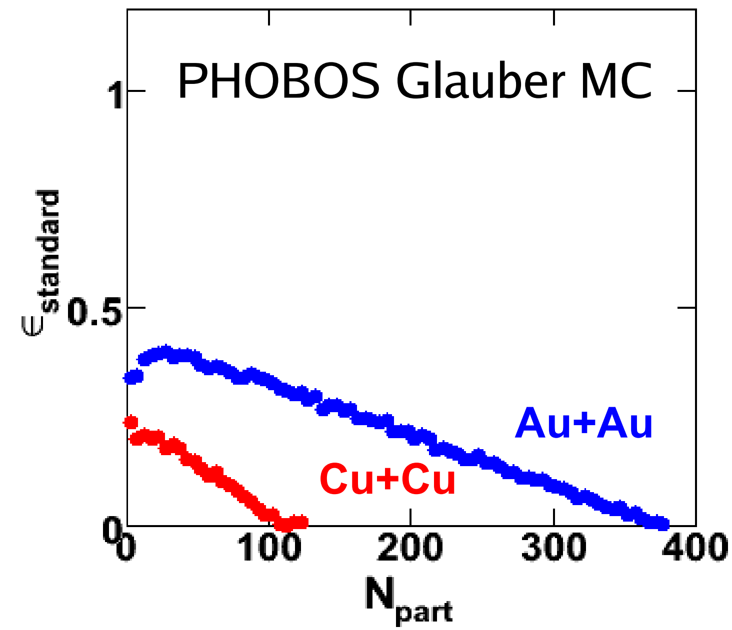


Large flow in central Cu+Cu compared to central Au+Au

Standard Eccentricity



$$\epsilon_{standard} = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$



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Cu+Cu, 200+62.4 GeV: nucl-ex/0610037 (sub.to PRL)
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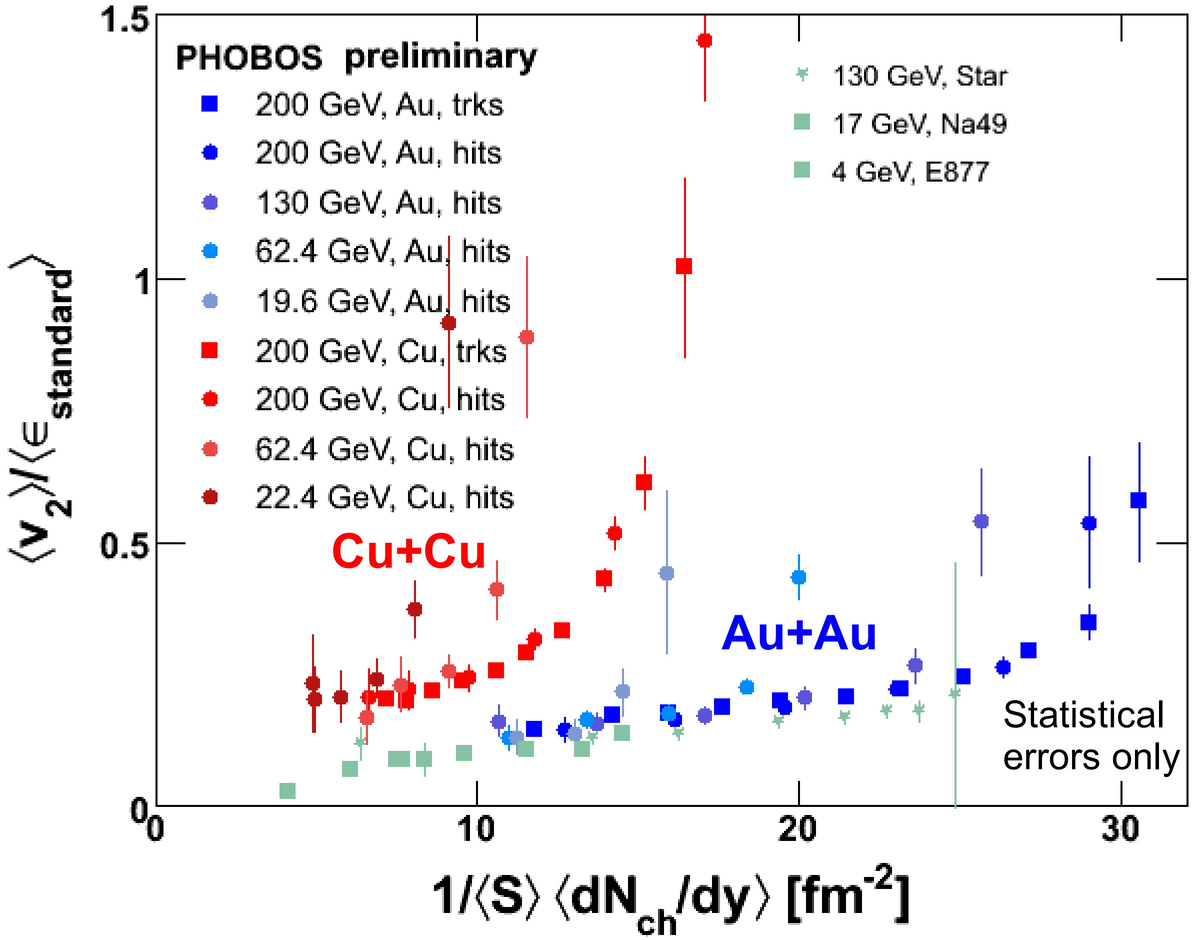
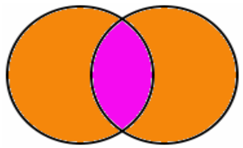
Elliptic flow scaled with $\epsilon_{standard}$





Small print:

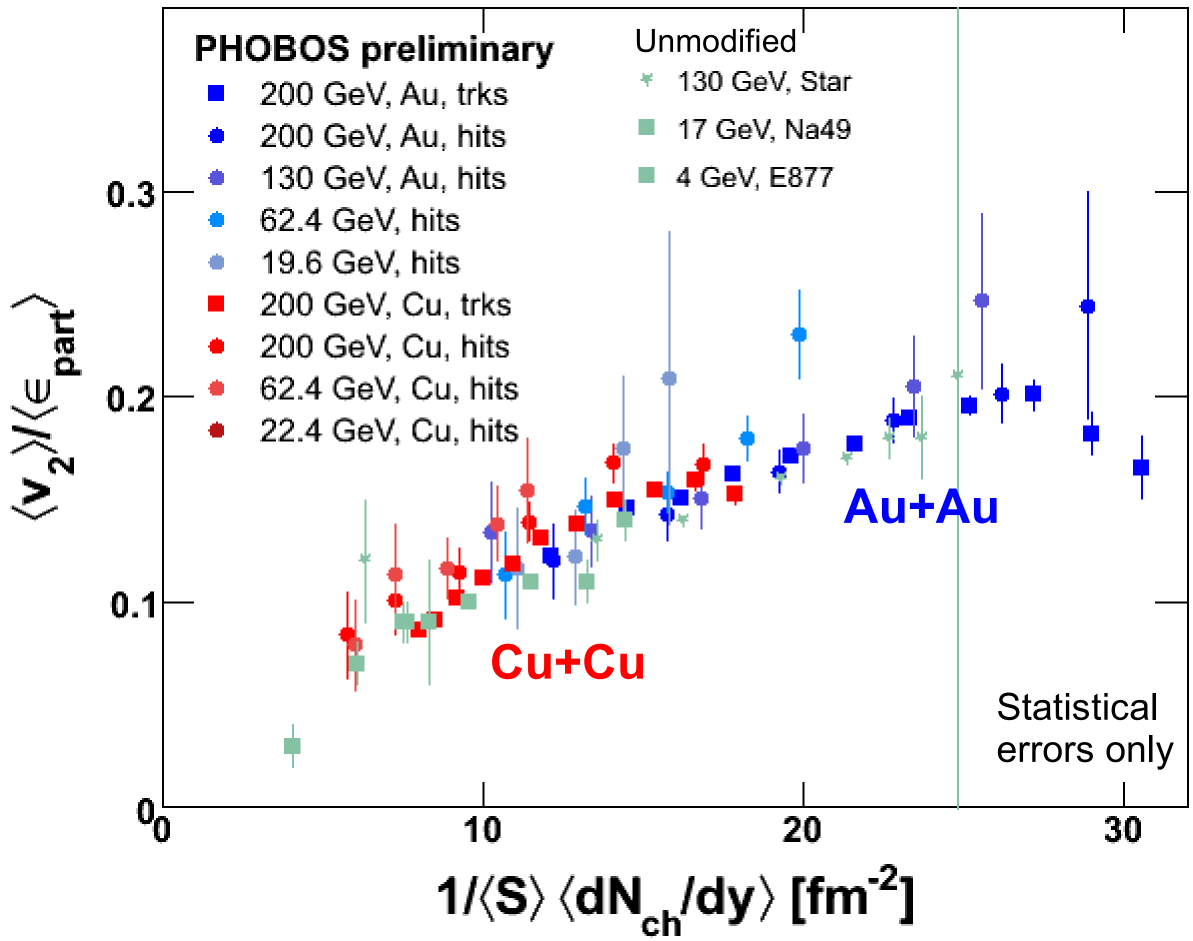
- Scale $v_2(\eta)$ to $v_2(y)$ (10% lower)
- Scale $dN/d\eta$ to dN/dy (15% higher)
- S is overlap area (MC Glauber)

No scaling between Cu+Cu and Au+Au

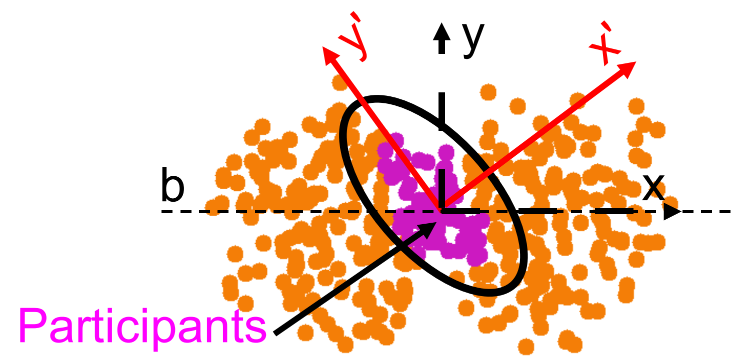
Au+Au, 200,130,62.4+19.6 GeV: PRL 94 122303 (2005)
Cu+Cu, 200+62.4 GeV: nucl-ex/0610037 (sub.to PRL)
Cu+Cu, 22.4 GeV: prel. QM06

STAR, PRC 66 034904 (2002)
Voloshin, Poskanzer, PLB 474 27 (2000)
Heiselberg, Levy, PRC 59 2716, (1999)

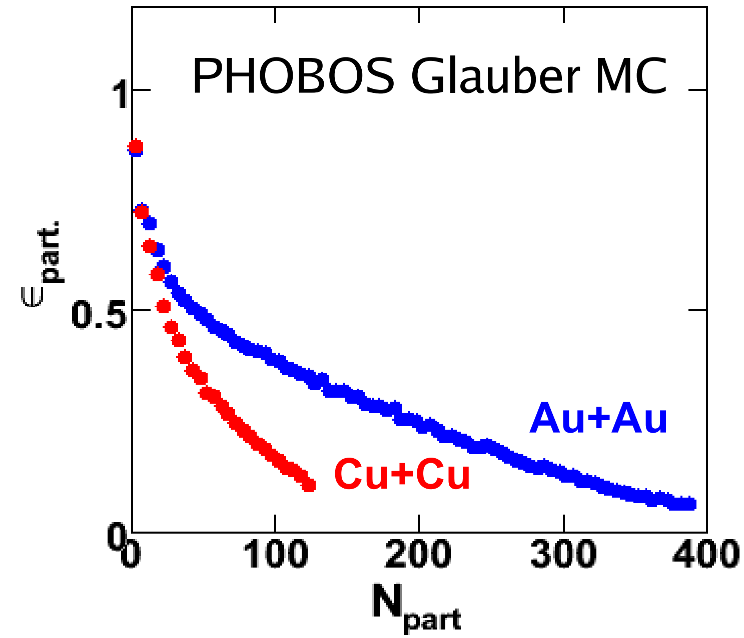
Elliptic flow and participant eccentricity



Participant Eccentricity



$$\epsilon_{part} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2}$$



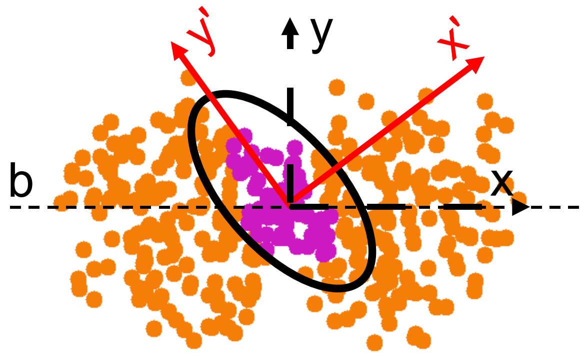
Approximate scaling between Cu+Cu and Au+Au

Au+Au, 200,130,62.4+19.6 GeV: PRL 94 122303 (2005)
Cu+Cu, 200+62.4 GeV: nucl-ex/0610037 (sub.to PRL)
Cu+Cu, 22.4 GeV: prel. QM06

Expected elliptic flow fluctuations

Elliptic flow seems to be developed event-by-event with respect to the overlap region

$$v_2 \sim \epsilon_{part}$$



$$\frac{\sigma_{v_2}}{\langle v_2 \rangle} \sim \frac{\sigma_{\epsilon_{part}}}{\langle \epsilon_{part} \rangle}$$

Expected elliptic flow fluctuations

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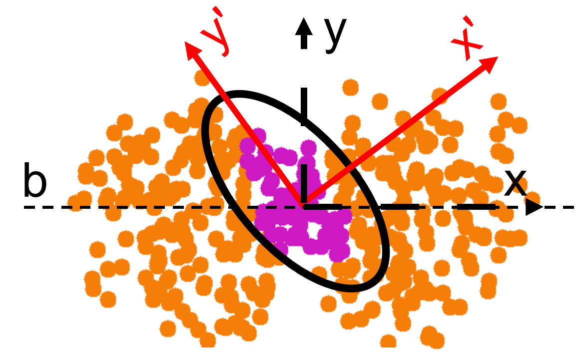
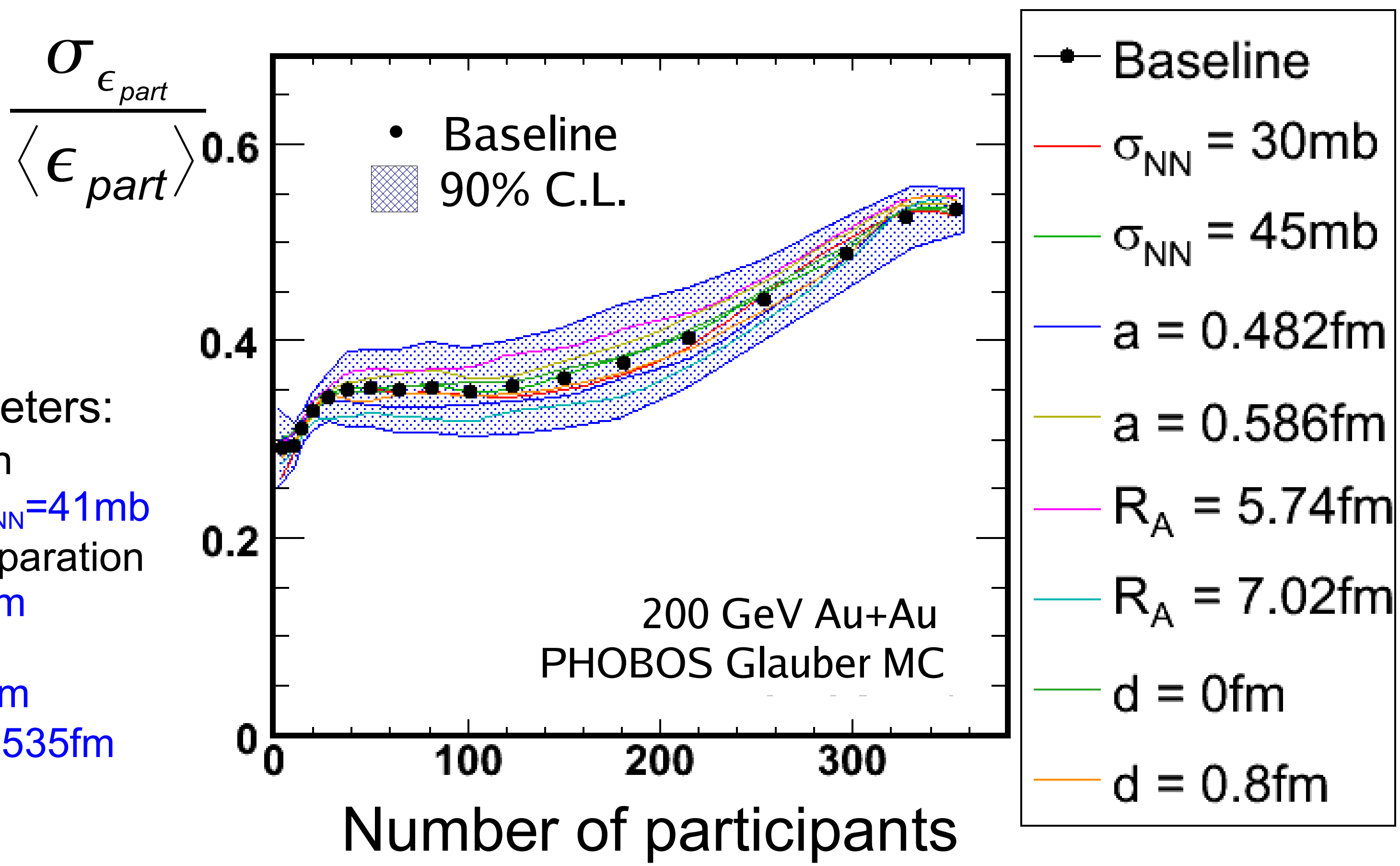
$$v_2 \sim \epsilon_{part}$$





Baseline parameters:

- Nucleon-nucleon cross section: σ_{NN}=41mb
- Inter-nucleon separation distance: d=0.4fm
- Wood-saxon radius: R_A=6.38fm
- Skin depth: a=0.535fm

Outline

First measurement of elliptic flow fluctuations

Method - 2 novel features

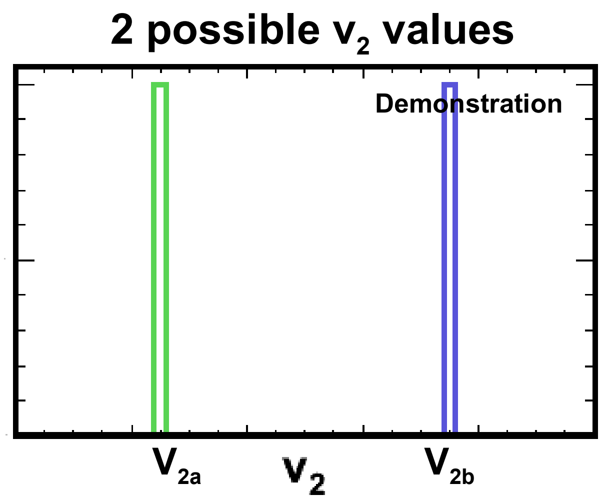
Event-by-event measurement technique developed for the PHOBOS detector

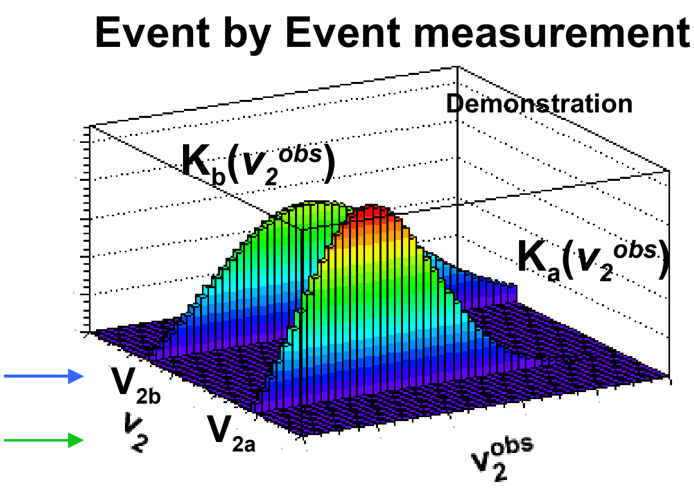
Extraction of dynamical fluctuations relying on the understanding of extensive MC simulations.

Results

Preliminary results presented at QM2006

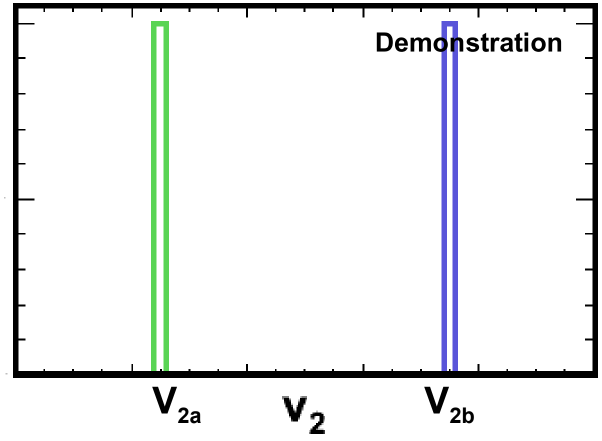
Method overview



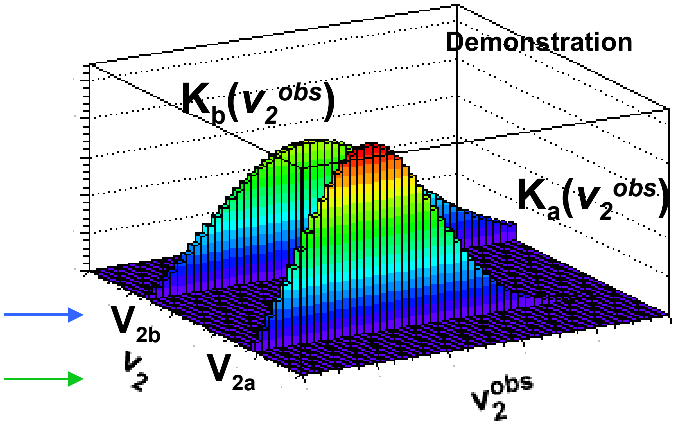


Method overview

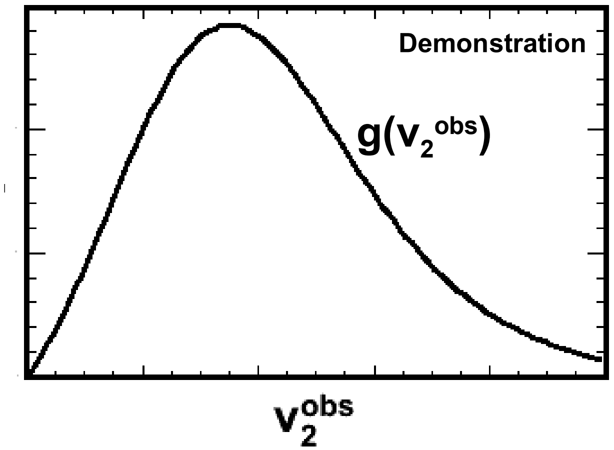
2 possible v_2 values



Event by Event measurement



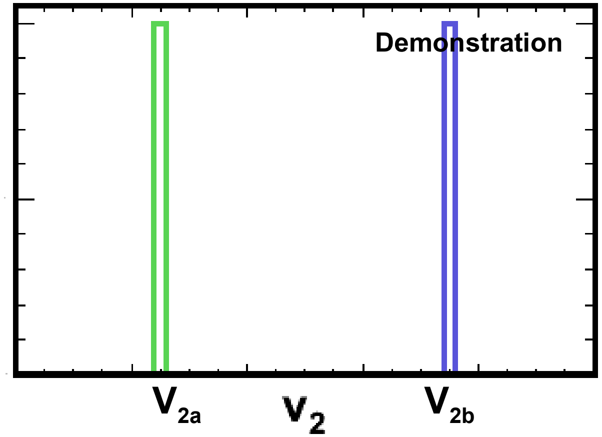
v_2^{obs} distribution in "data"



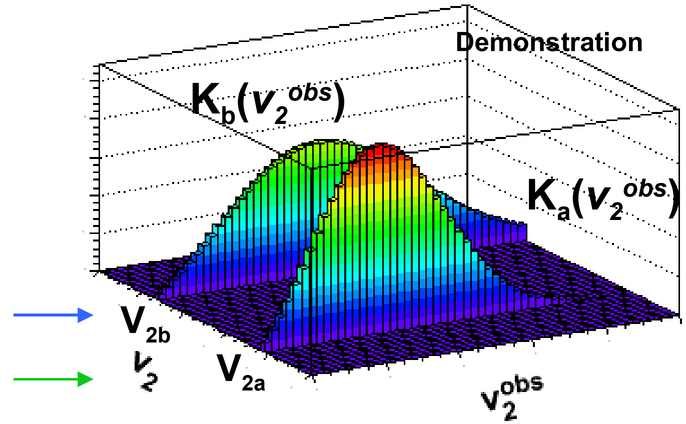
Question: What is the relative abundance of 2 v_2's in "data"?

Method overview

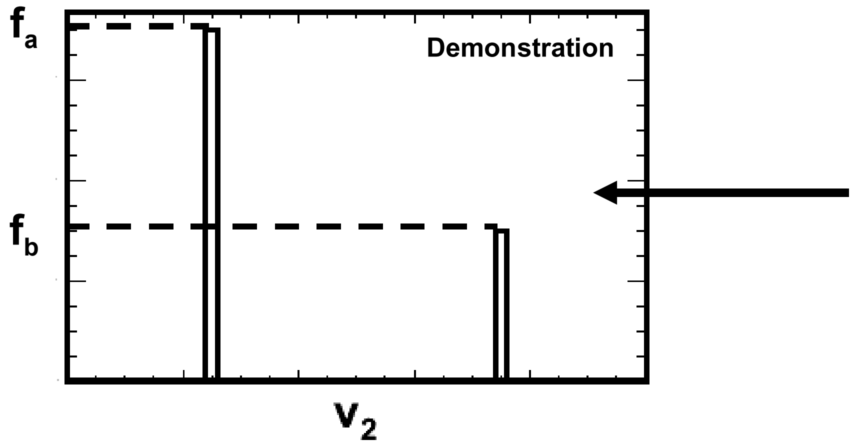
2 possible v_2 values



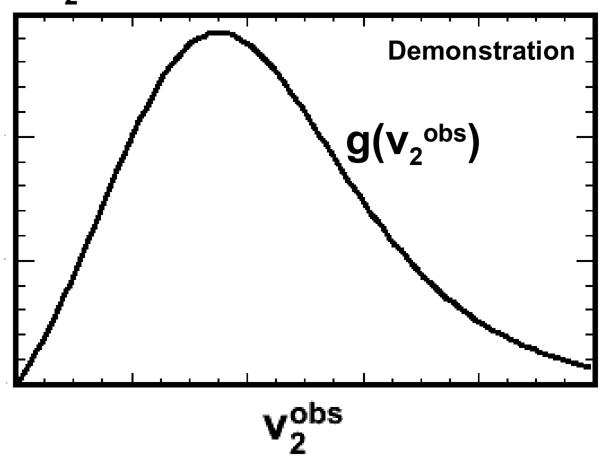
Event by Event measurement



Relative abundance in “data”



v_2^{obs} distribution in “data”



Question: What is the relative abundance of 2 v_2’s in “data”?

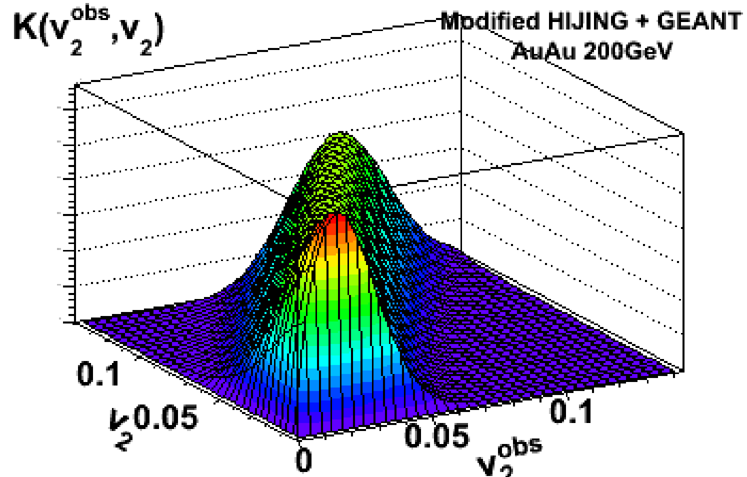
$$g(v_2^{obs}) = f_a K_a(v_2^{obs}) + f_b K_b(v_2^{obs})$$

Method overview

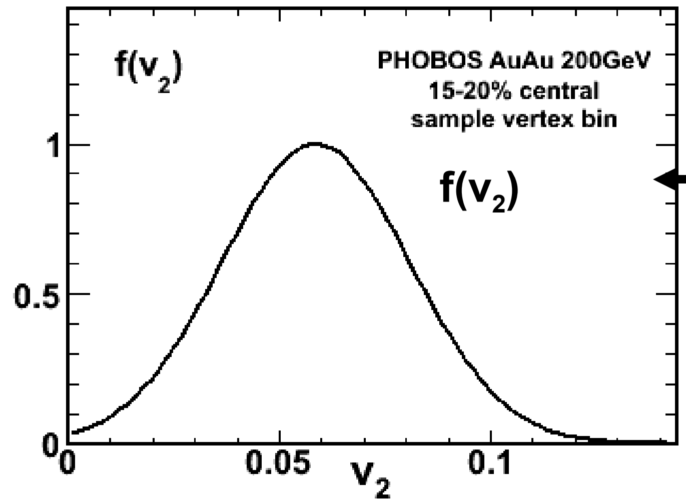
In real life v_2 can take a continuum of values

$$g(v_2^{obs}) = \int_0^1 K(v_2^{obs}, v_2) f(v_2)\, dv_2$$

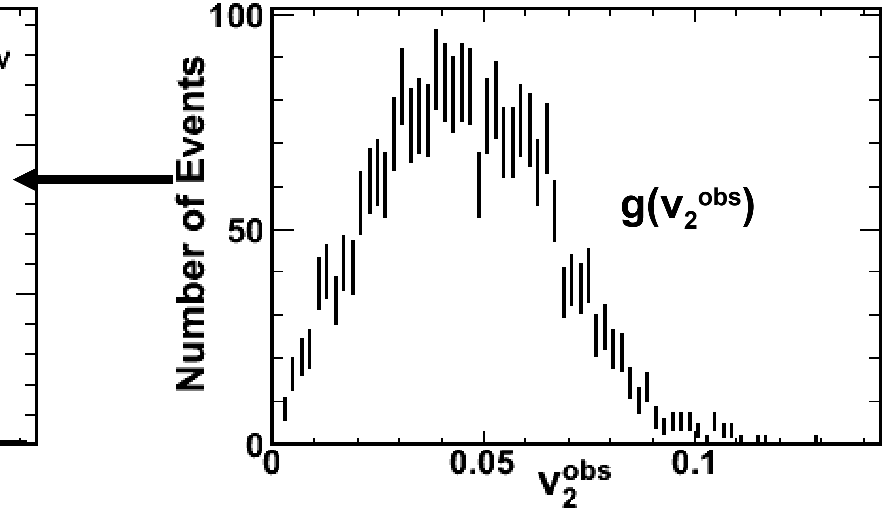
Kernel – Response Function



Extracted true v_2 distribution



v_2^{obs} distribution in "data"



Method Overview

If $K(v_2^{obs}, v_2) = \exp\left(\frac{-(v_2^{obs} - v_2)^2}{2\sigma_{stat}^2}\right)$ **Then** $\sigma^2 = \sigma_{dyn}^2 + \sigma_{stat}^2$

Method Overview

If $K(v_2^{obs}, v_2) = \exp\left(\frac{-(v_2^{obs} - v_2)^2}{2\sigma_{stat}^2}\right)$ **Then** $\sigma^2 = \sigma_{dyn}^2 + \sigma_{stat}^2$

However $K(v_2^{obs}, v_2, n) = \frac{v_2^{obs}}{\sigma^2} e^{-\left(\frac{v_2^{obs} + v_2^2}{2\sigma^2}\right)} I_0\left(\frac{-v_2^{obs} v_2}{\sigma^2}\right)$

Method Overview

However $K(v_2^{obs}, v_2, n) = \frac{v_2^{obs}}{\sigma^2} e^{-\left(\frac{v_2^{obs} + v_2^2}{2\sigma^2}\right)} I_0\left(\frac{-v_2^{obs} v_2}{\sigma^2}\right)$

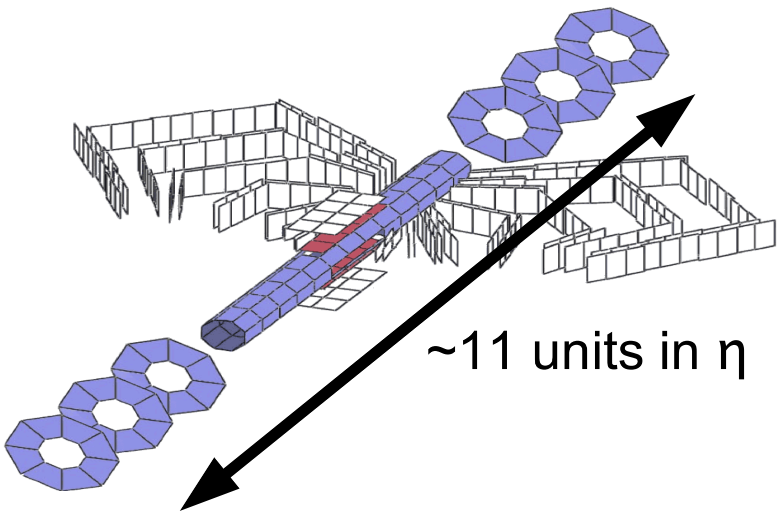
The analysis has 3 main steps:

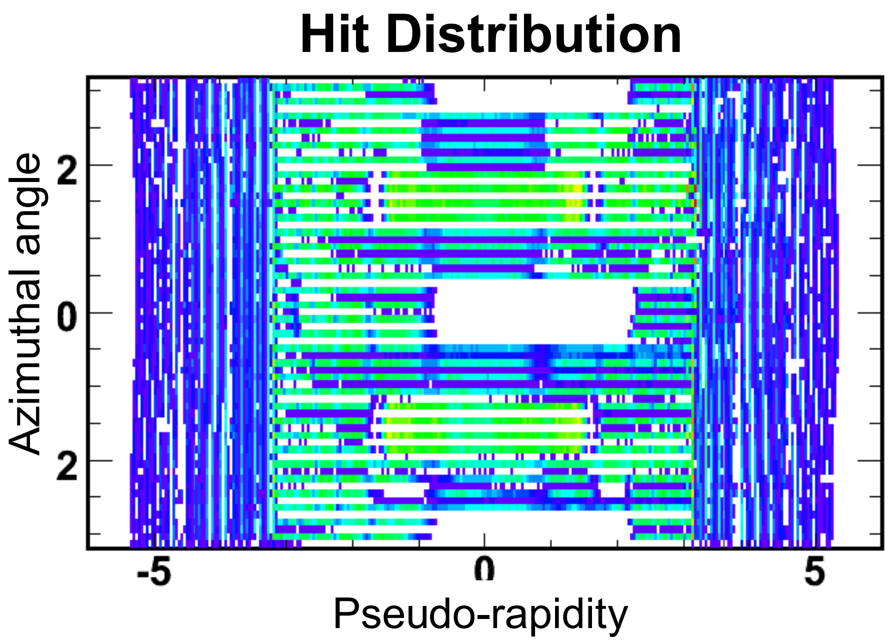
- **Measuring v_2^{obs} event-by-event in data: $g(v_2^{obs})$**
- **Calculating the Kernel: $K(v_2^{obs}, v_2)$**
- **Extracting dynamical fluctuations: $f(v_2)$**

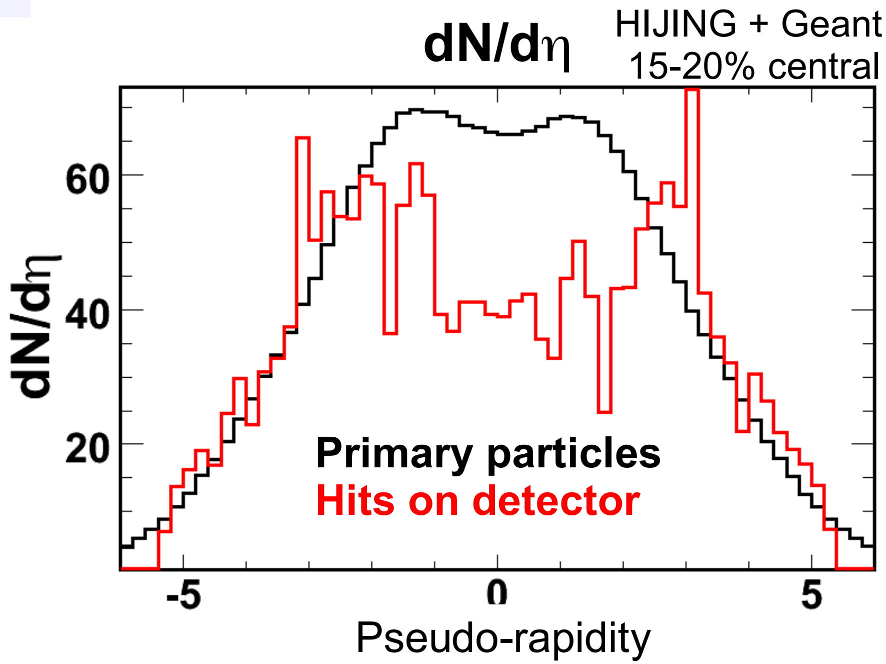
$$g(v_2^{obs}) = \int_0^1 K(v_2^{obs}, v_2) f(v_2) dv_2$$

Event-by-event measurement of v_2^{obs}

- PHOBOS Multiplicity Array
  - $-5.4<\eta<5.4$ coverage
  - Holes and granularity differences
- Usage of all available information
  in event to determine event-by-event a single value for v_2^{obs}







Event-by-event measurement of v_2^{obs}

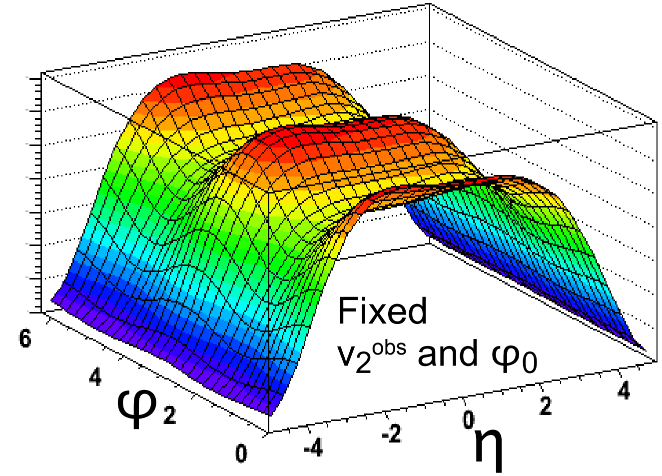
Define probability distribution function (PDF)for hit positions:

$$P(\eta,\phi;\, v_2^{obs},\phi_0)=\frac{1}{s(v_2^{obs},\phi_{0,}\eta)}[1+2v_2(\eta)\cos(2\phi-2\phi_0)]$$

Normalization

Normalization assures integral of PDF folded with the acceptance is the same for different values of v_2^{obs} and ϕ_0.

Probability distribution function



$$s(v_2^{obs},\phi_0;\eta)=\int A(\eta,\phi)[1+2v_2(\eta)\cos(2\phi-2\phi_0)]\,d\phi$$

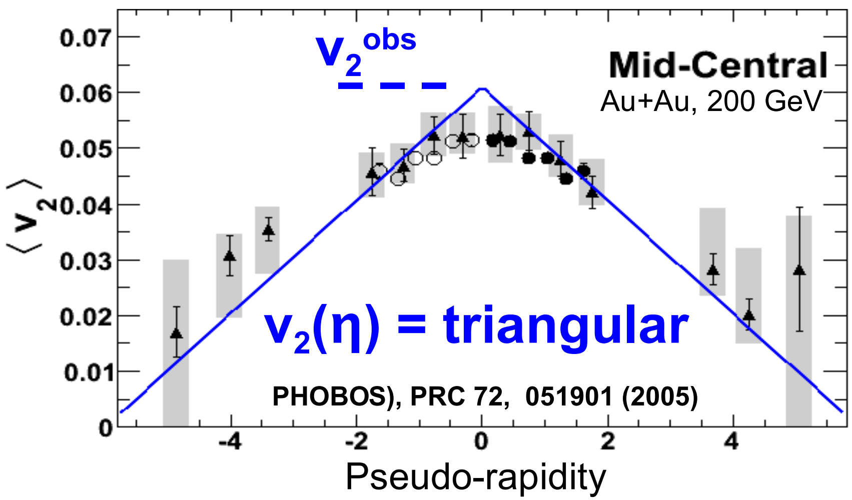
Acceptance

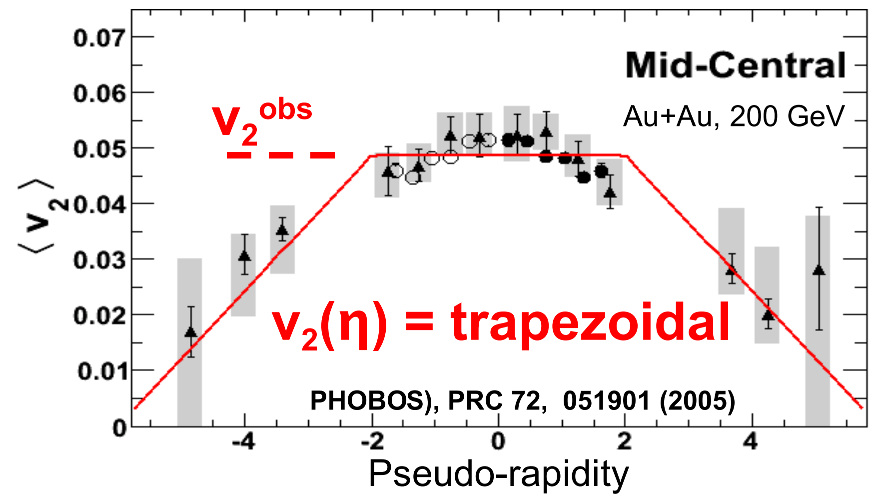
Event-by-event measurement of v_2^{obs}

Define probability distribution function (PDF)for hit positions:

$$P(\eta,\phi;\, v_2^{obs},\phi_0)=\frac{1}{s(v_2^{obs},\phi_{0,}\eta)}[1+2v_2(\eta)\cos(2\phi-2\phi_0)]$$

We parameterize $v_2(\eta)$ using known shape from previous measurements:





Event-by-event measurement of v_2^{obs}

Define probability distribution function (PDF)for hit positions:

$$P(\eta,\phi;\ v_2^{obs},\phi_0)=\frac{1}{s(v_2^{obs},\phi_{0,}\eta)}[1+2v_2(\eta)\cos(2\phi-2\phi_0)]$$

For a given event with n hits, the likelihood of v_2^{obs} and ϕ_0:

$$L(v_2^{obs},\ \phi_0)=\prod_{i=1}^{n} P(\eta_i,\phi_i;\ v_2^{obs},\phi_0)$$

Maximizing L allows a measurement of v_2^{obs} and ϕ_0 event-by-event.

Determining the kernel

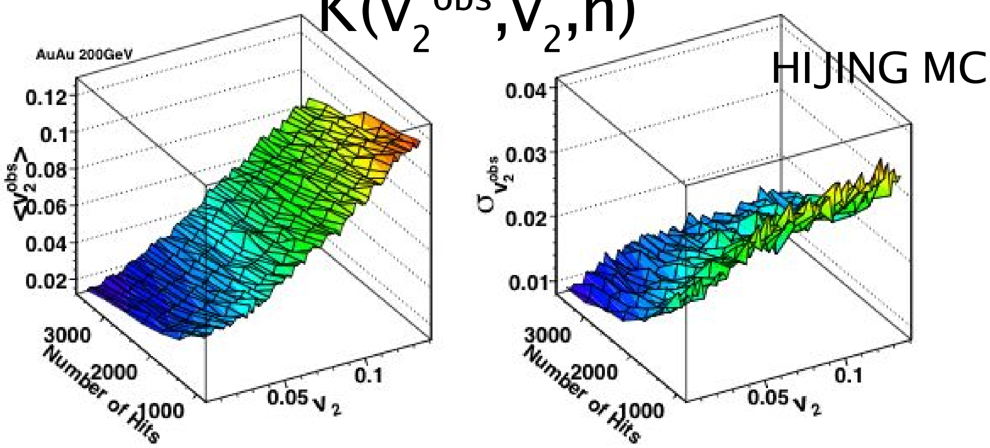
Reminder: Kernel is the response of the measurement to input value of v_2.

$$g(v_2^{obs}) = \int_0^1 K(v_2^{obs}, v_2)\, f(v_2)\, dv_2$$

Response also depends on the observed multiplicity n.

Determining the kernel = "measuring" v_2^{obs} distributions in MC in bins of v_2 and n.

$K(v_2^{obs}, v_2, n)$



$1.5 \cdot 10^6$ HIJING events
Modified φ to include
triangular or trapezoidal flow

Determining the kernel

Fitting $K(v_2^{obs},v_2,n)$ with smooth functions reduces bin-to-bin fluctuations.

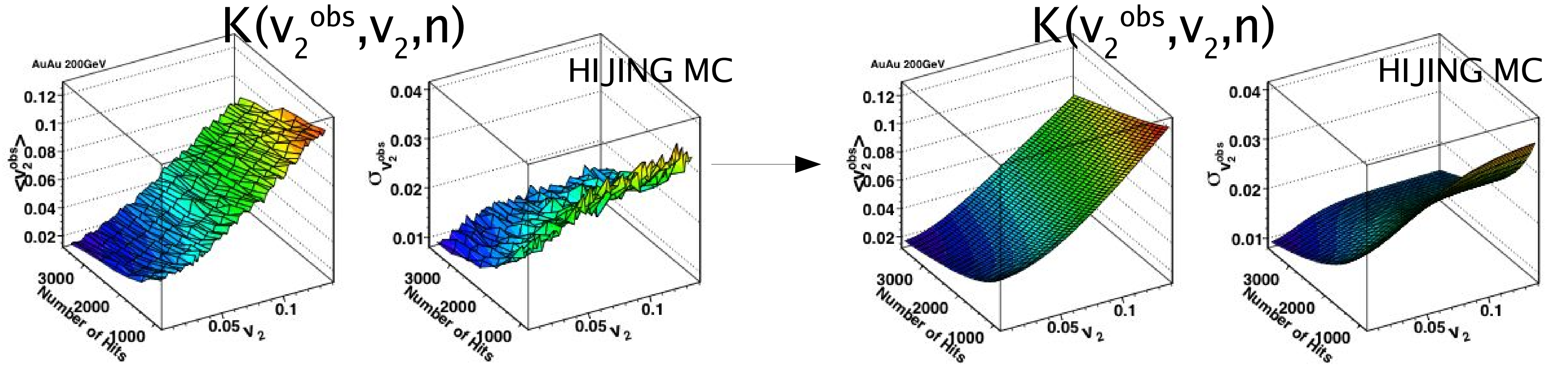
Theoretical distribution of $K(v_2^{obs},v_2,n)$ modified for experimental effects is used as fit function:

$$K(v_2^{obs},v_2,n)=\frac{v_2^{obs}}{\sigma^2}\,e^{-\left(\frac{v_2^{obs}+v_2^2}{2\sigma^2}\right)}\,I_0\left(\frac{-v_2^{obs}v_2}{\sigma^2}\right)$$

(J.-Y.Ollitrault, PRD (1992) 46, 226)

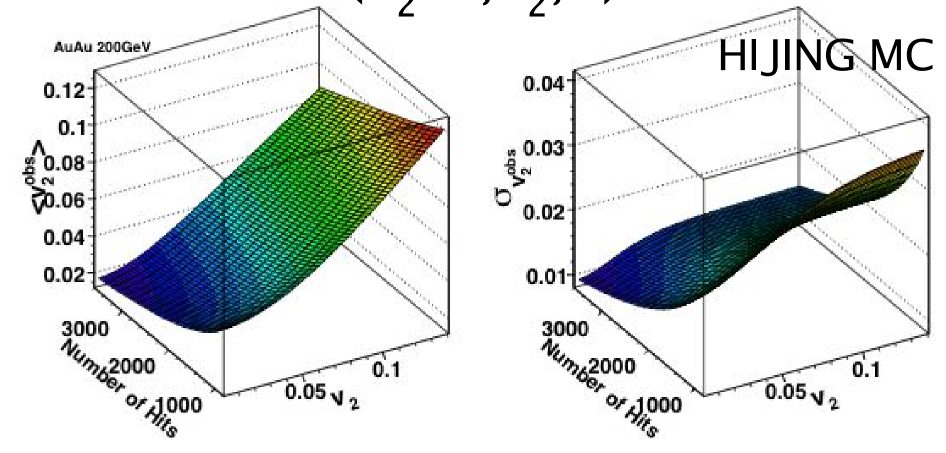
$$v_2\rightarrow(An+B)v_2 \quad \text{(suppression)}$$

$$\sigma=\frac{C}{\sqrt{n}}+D \quad \text{(finite resolution)}$$



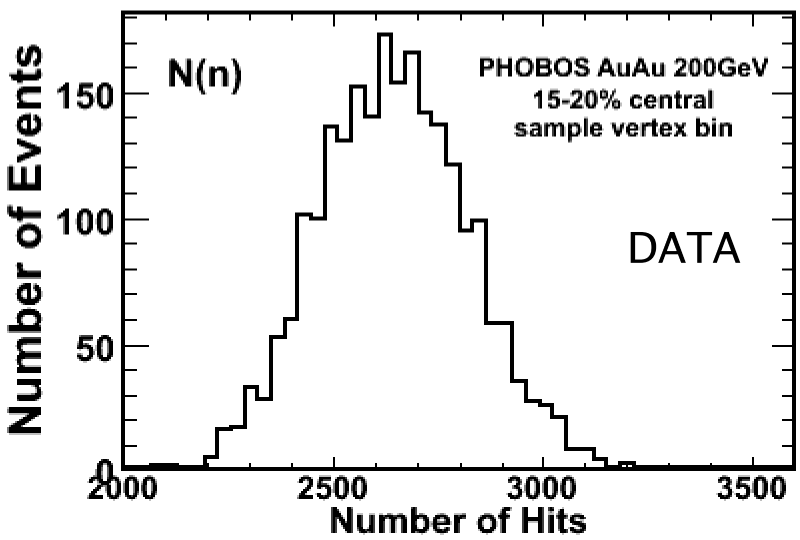
Determining the kernel

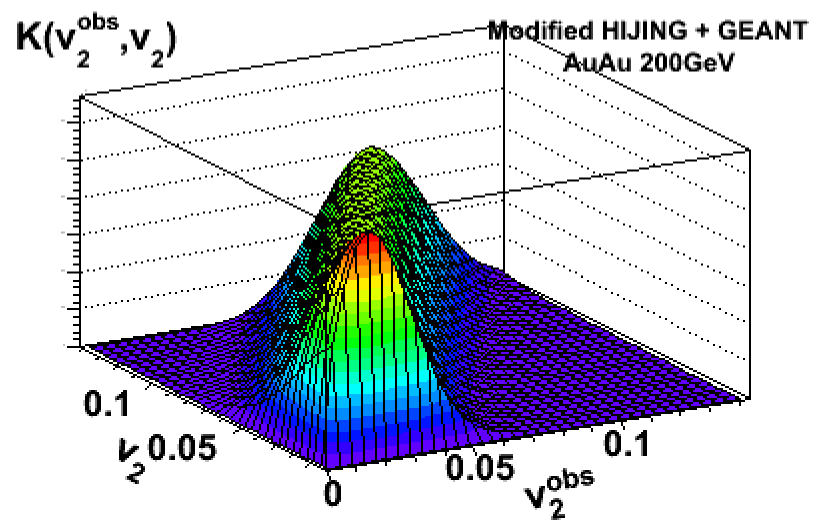
$K(v_2^{obs}, v_2, n)$



Assuming that the true v_2 distribution for a set of events in a given centrality class is independent of n, it is possible to integrate out the multiplicity dependence:

$$K(v_2^{obs}, v_2) = \int K(v_2^{obs}, v_{2,} n) N(n) dn$$





Extracting dynamical fluctuations

$$\underline{g\left(v_2^{obs}\right)} = \int_0^1 \underline{K\left(v_2^{obs}, v_2\right)}\, \underline{f\left(v_2\right)}\, dv_2$$

Measured

Constructed from MC

Extracting dynamical fluctuations

$$\underbrace{g(v_2^{obs})}_{\text{Measured}} = \int_0^1 \underbrace{K(v_2^{obs}, v_2)}_{\text{Constructed from MC}} \underbrace{f(v_2)}_{} dv_2$$

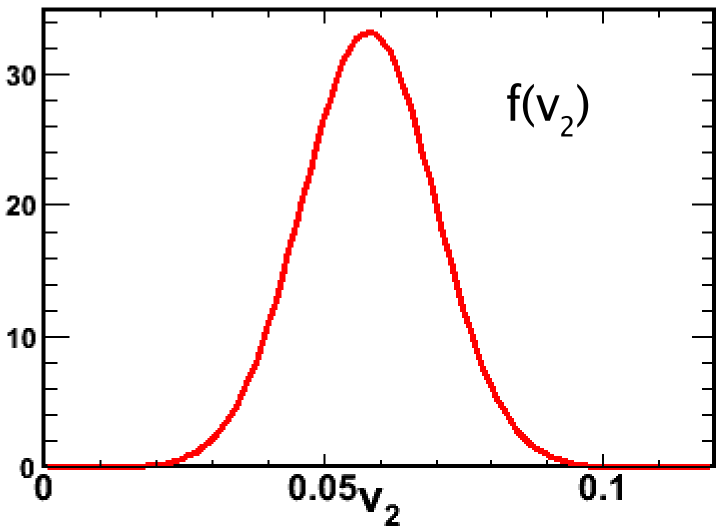
Measured

Constructed from MC

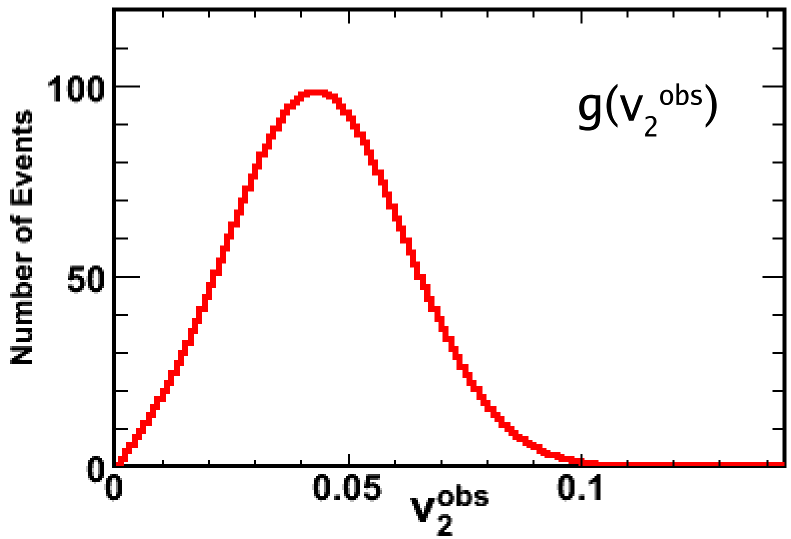
Gaussian Ansatz:

$$f(v_2) = \exp\left[\frac{-(v_2 - \langle v_2 \rangle)^2}{2\sigma_{v_2}^2}\right]$$

Ansatz: true v_2 distribution



Expected $g(v_2^{obs})$ for ansatz



Extracting dynamical fluctuations

$$g\left(v_2^{obs}\right)=\int_0^1 K\left(v_2^{obs},v_2\right)f\left(v_2\right)dv_2$$

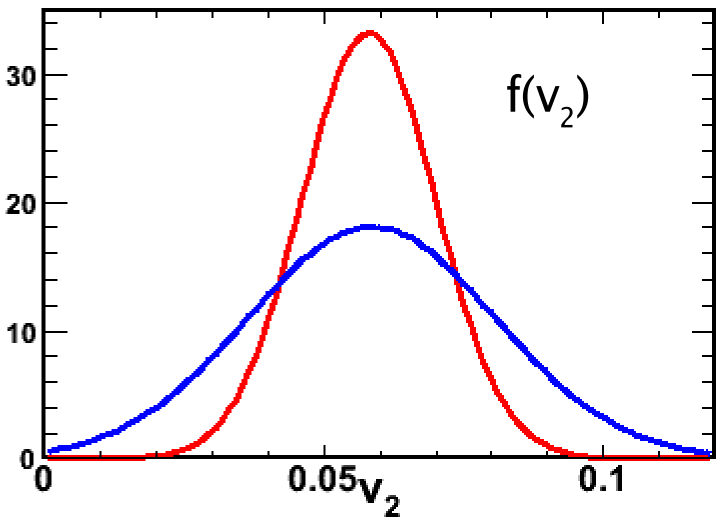
Measured — $g(v_2^{obs})$

Constructed from MC — $K(v_2^{obs}, v_2)$

Gaussian Ansatz: — $f(v_2)$

$$f\left(v_2\right)=\exp\left[\frac{-\left(v_2-\langle v_2\rangle\right)^2}{2\sigma_{v_2}^2}\right]$$

Ansatz: true v_2 distribution



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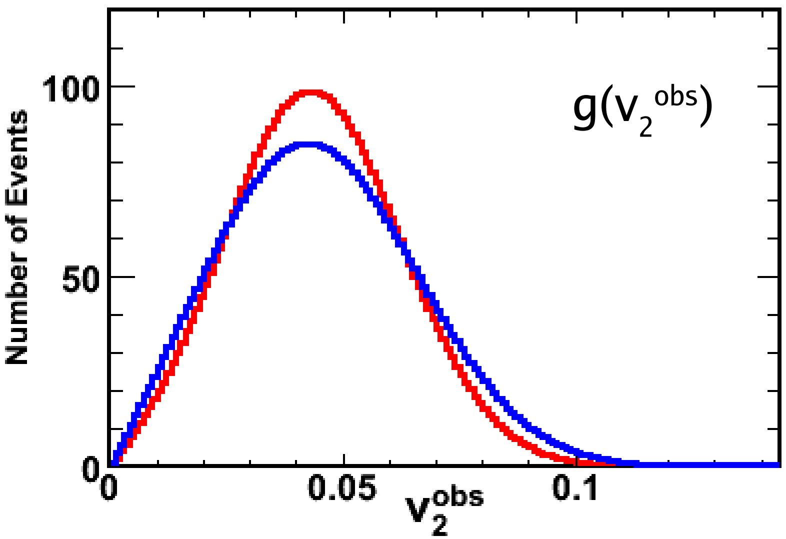

Extracting dynamical fluctuations

$$g(v_2^{obs}) = \int_0^1 K(v_2^{obs}, v_2) f(v_2) dv_2$$

Measured — $g(v_2^{obs})$

Constructed from MC — $K(v_2^{obs}, v_2)$

Gaussian Ansatz:

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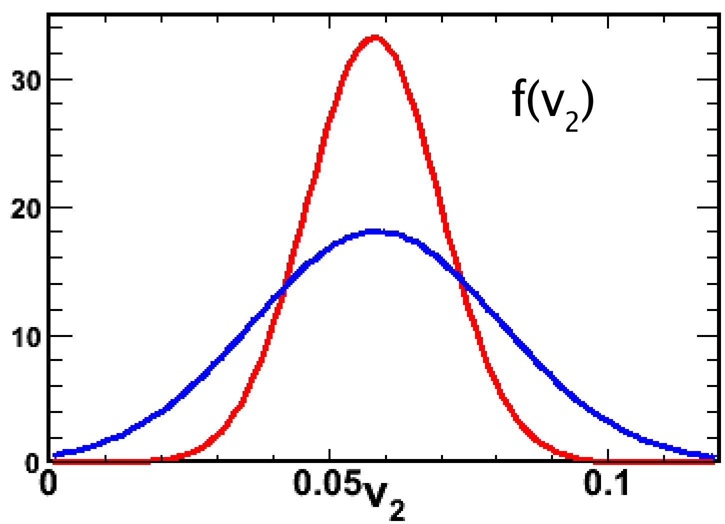
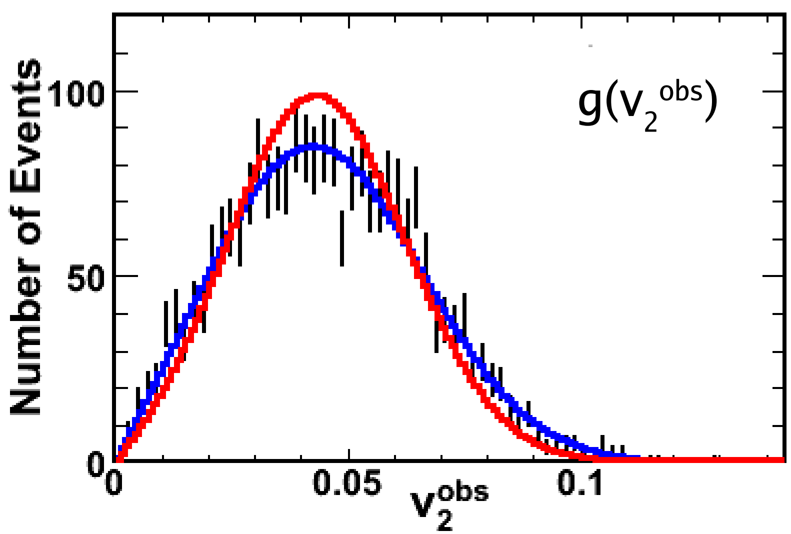
Ansatz: true v_2 distribution



Comparison with data



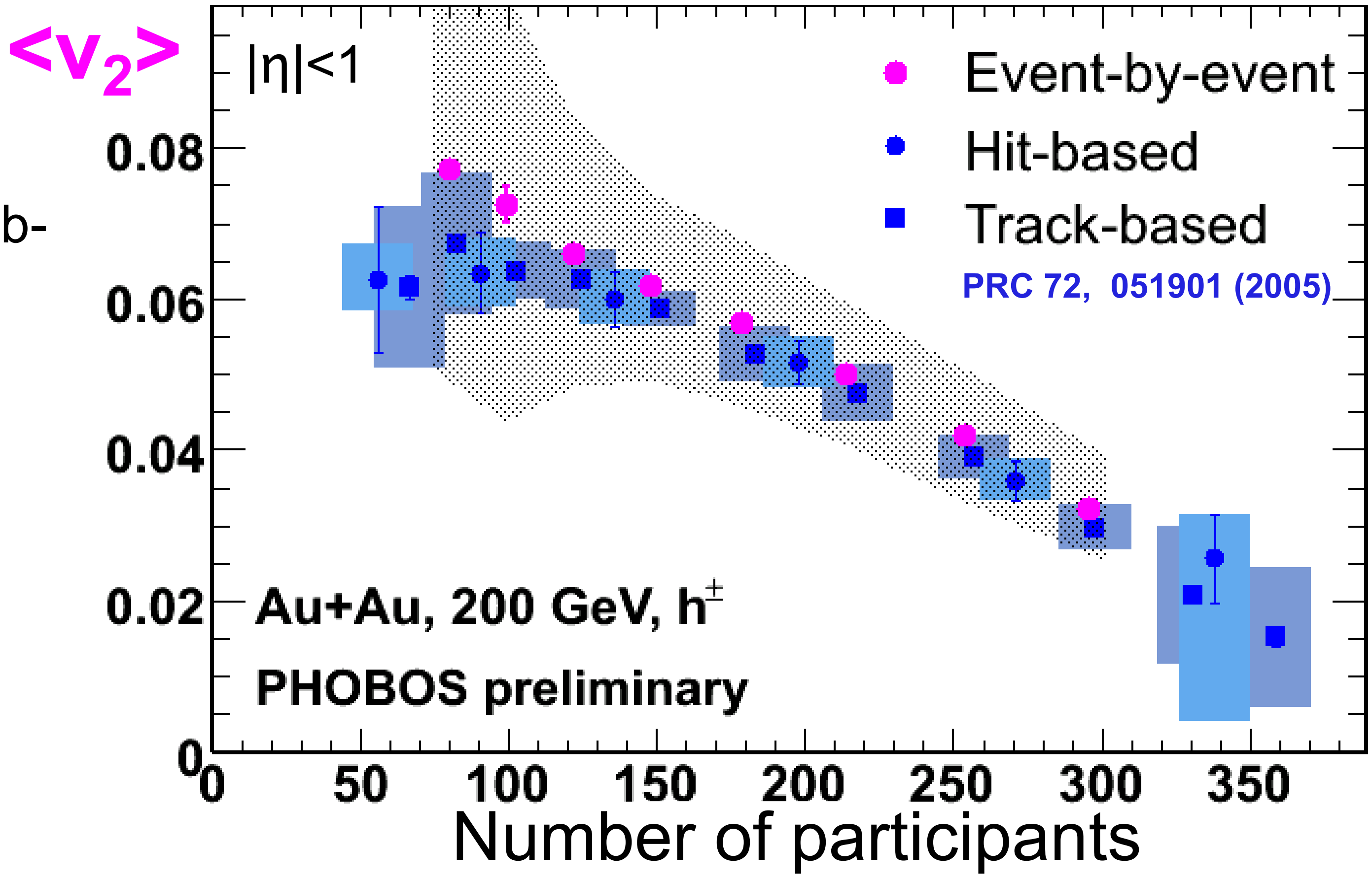
Compare expected $g(v_2^{obs})$ for trials with data:

Maximum-Likelihood fit → $\langle v_2 \rangle$ and σ_{v_2}

Event-by-event mean v_2 vs published results

- Standard methods
  - Hit- and track-based
  - Use reaction plane sub-even technique
- Event-by-event:
  - PR04 Au+Au data
    - No magnetic field
    - 500.000 events
    - 10 vertex bins ($-10cm < z_{vertex} < 10cm$)
  - Relate v_2^{obs} to $\langle v_2 \rangle$:

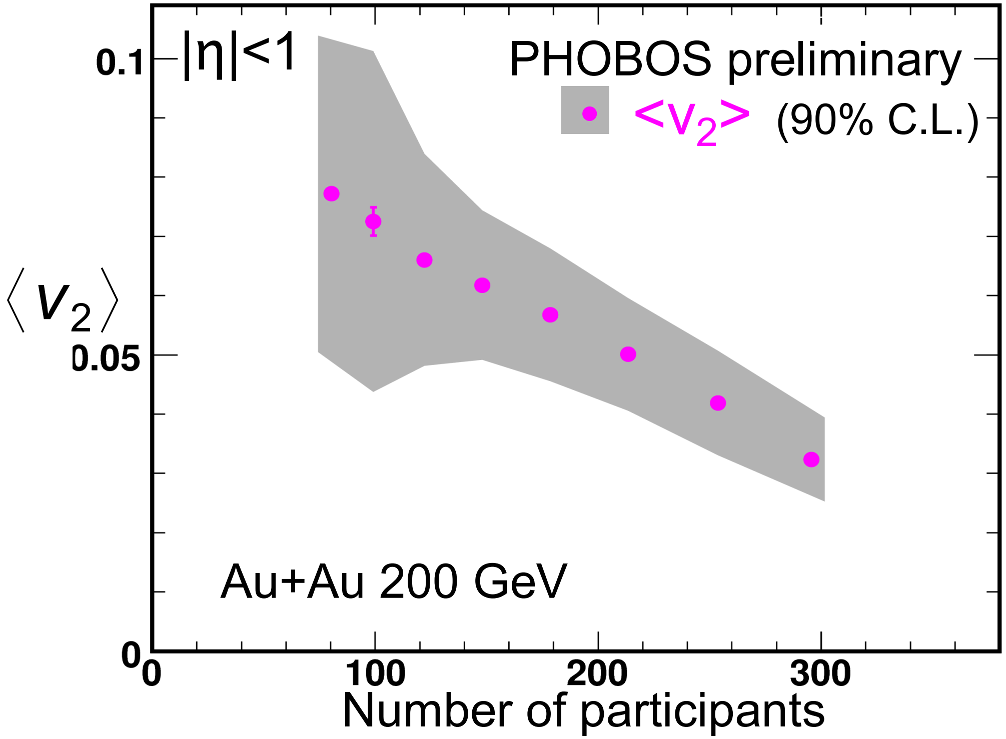
$$\langle v_2 \rangle(|\eta|<1) = 0.5 \times (11/12 \langle v_2^{triangular} \rangle + \langle v_2^{trapezodial} \rangle)$$



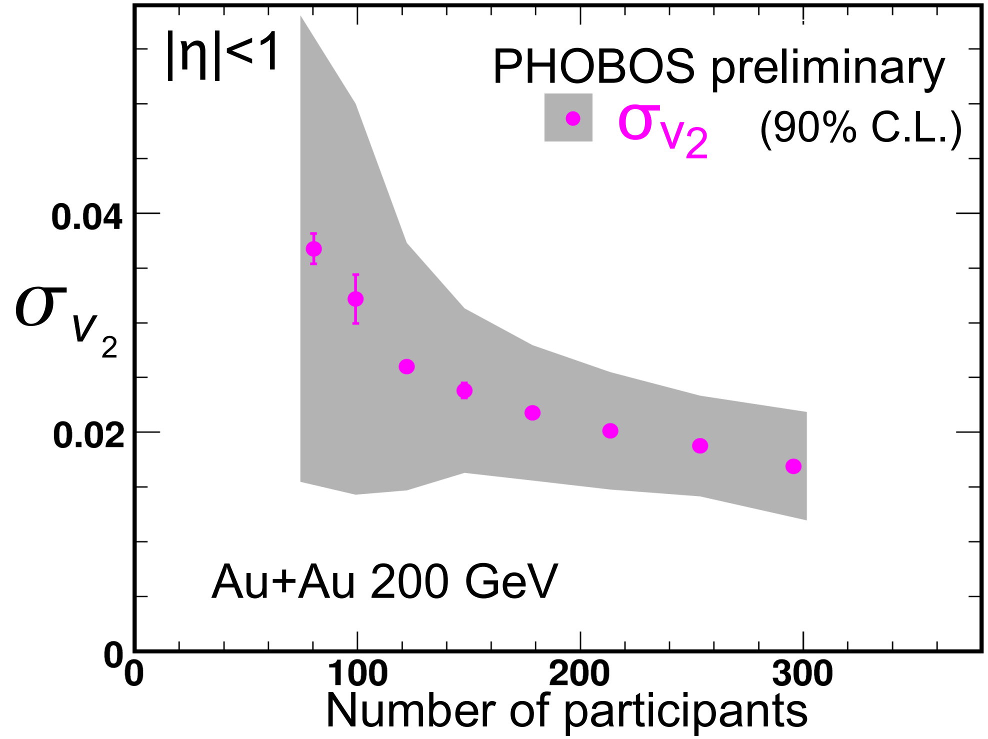
Very good agreement of the event-by-event measured v_2 with the hit- and tracked-based published results

Elliptic flow fluctuations: $\langle v_2 \rangle$ and σ_{v_2}

Mean elliptical flow



Dynamical flow fluctuations

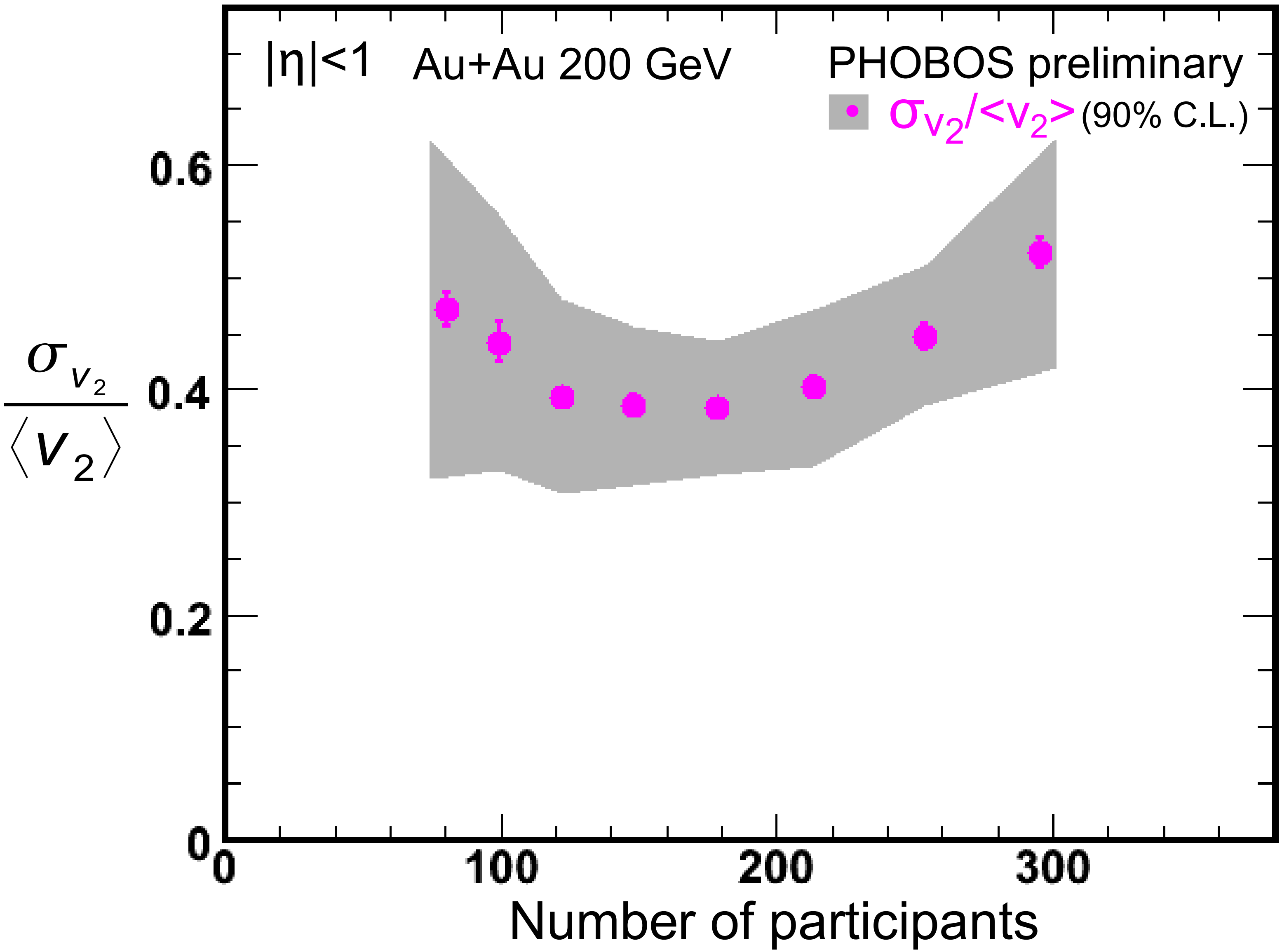


<u>Systematic errors:</u>
- Variation in η-shape
- Variation of $f(v_2)$
- MC response
- Vertex binning
- Φ_0 binning

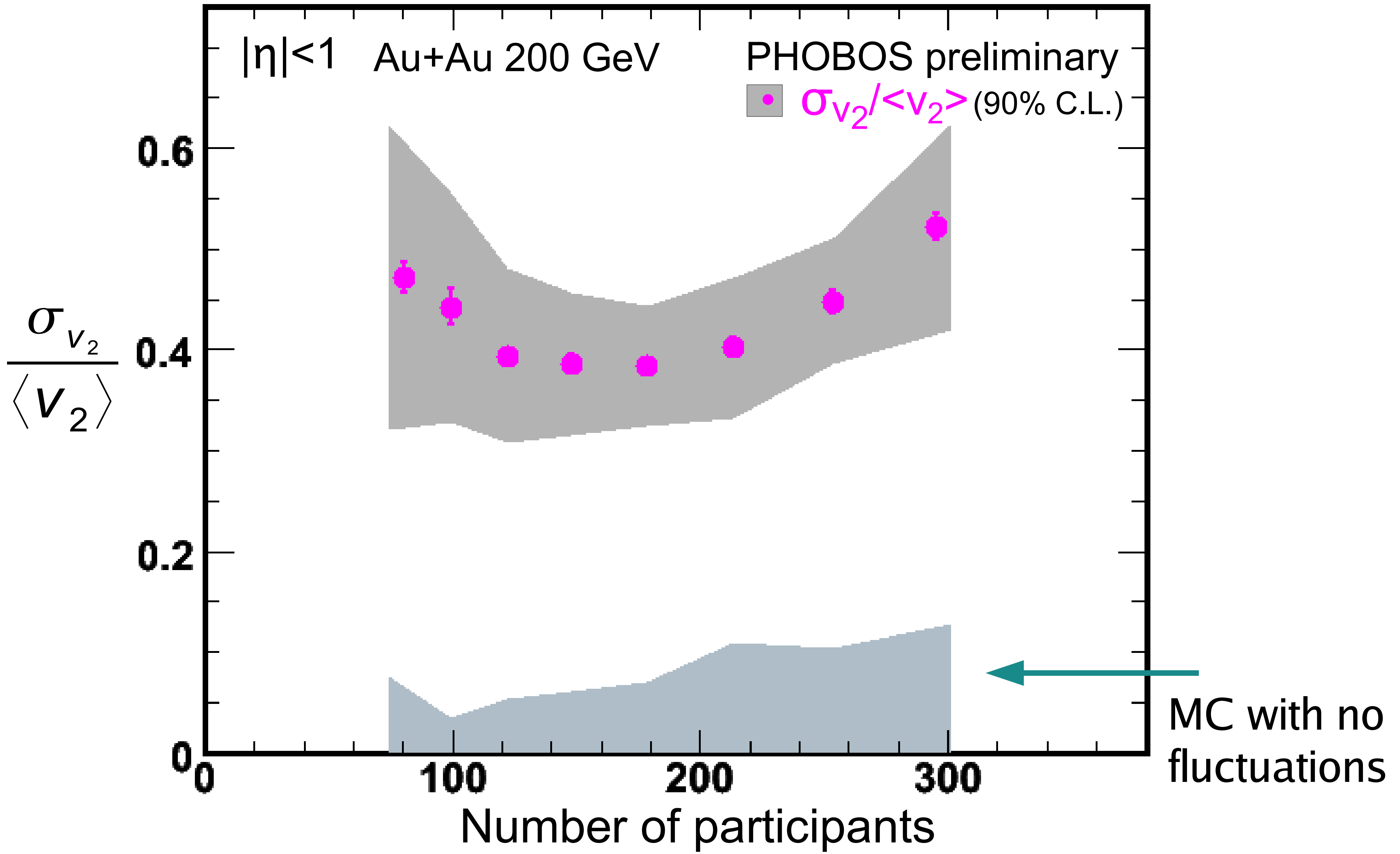
"Scaling" errors cancel in the ratio: relative fluctuations, $\sigma_{v_2}/\langle v_2 \rangle$

see next slide

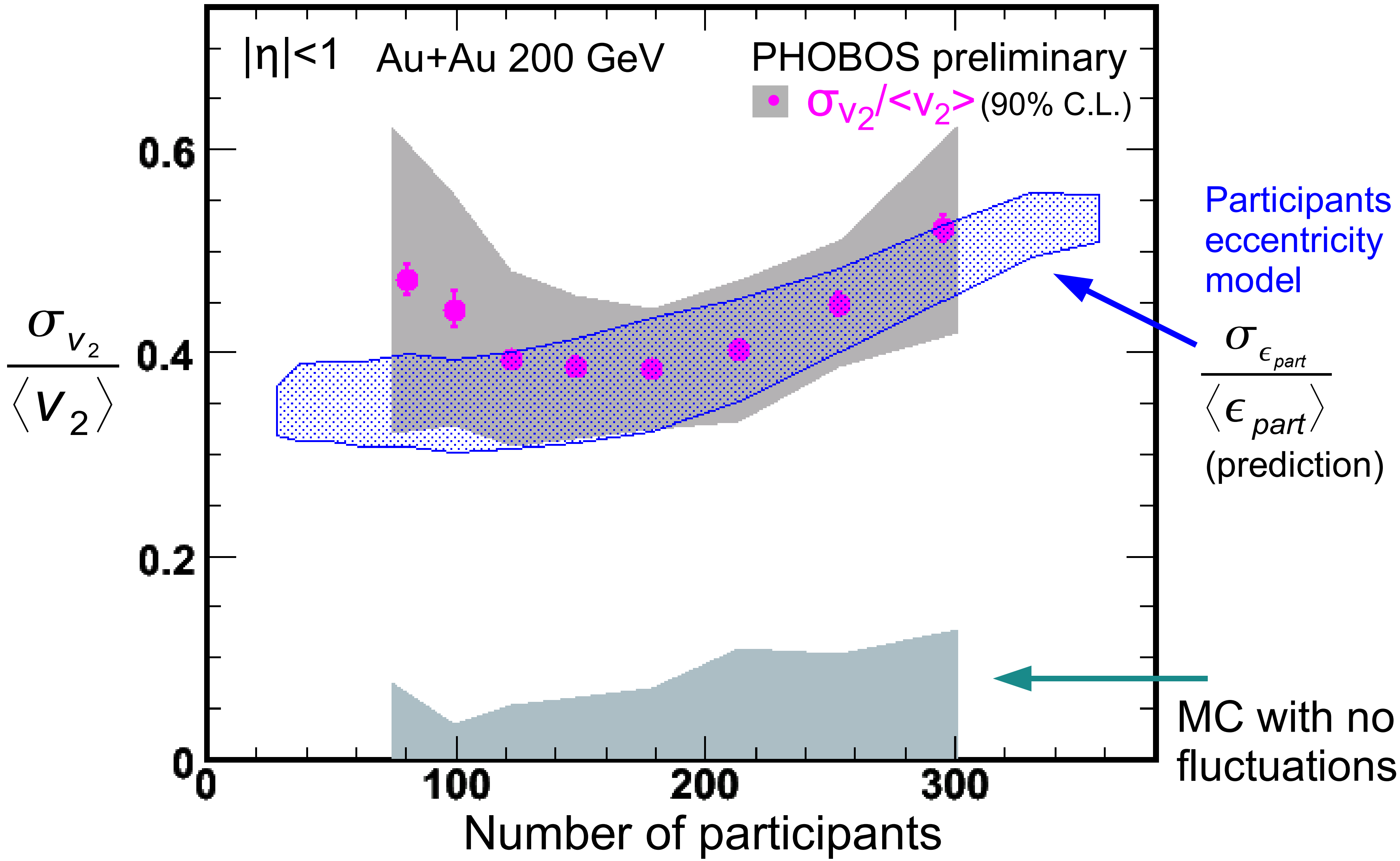
Elliptic flow fluctuations: $\sigma_{v_2}/\langle v_2 \rangle$



Elliptic flow fluctuations: $\sigma_{v_2}/\langle v_2\rangle$



Participant eccentricity compared to data

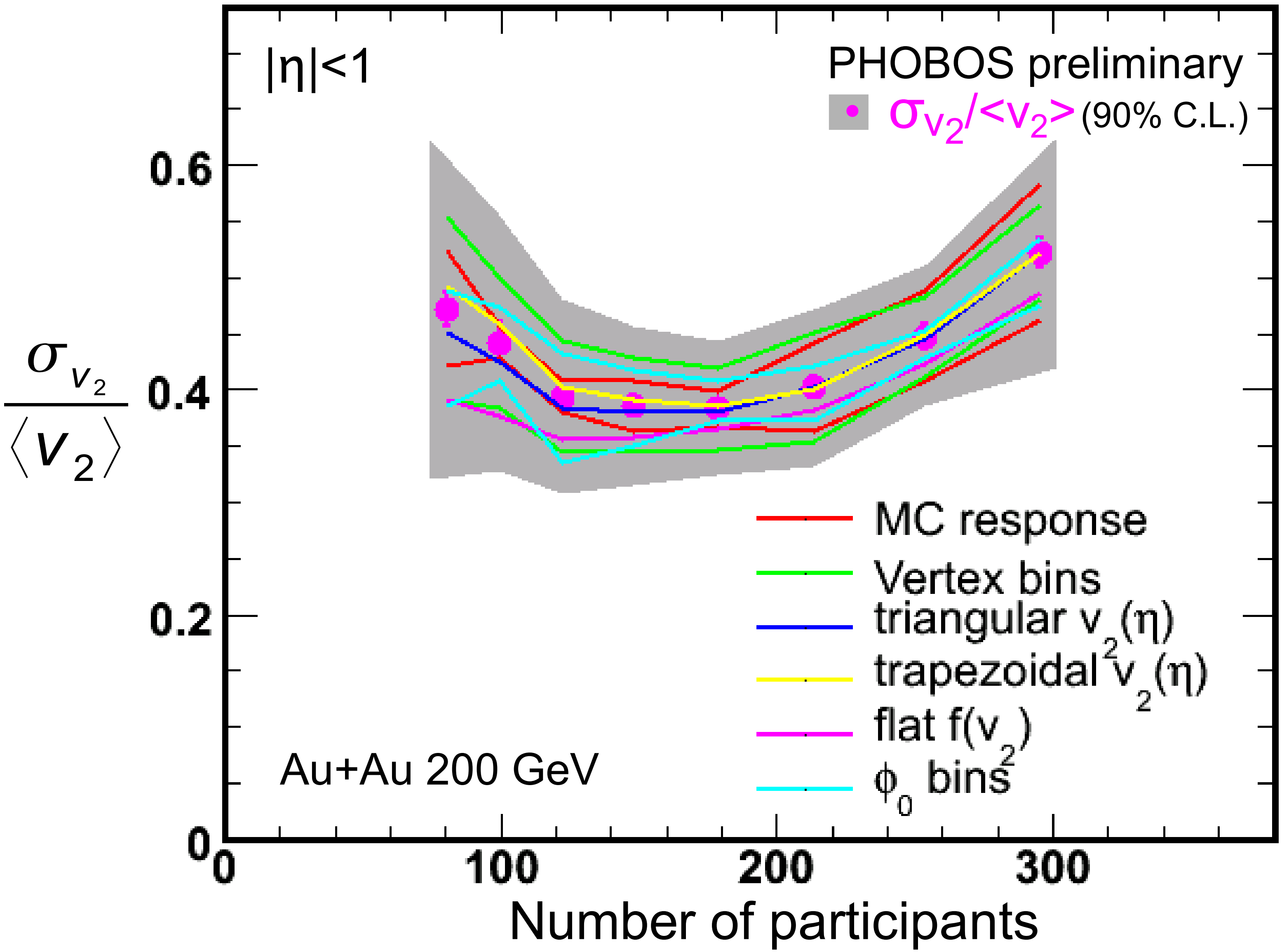


Summary

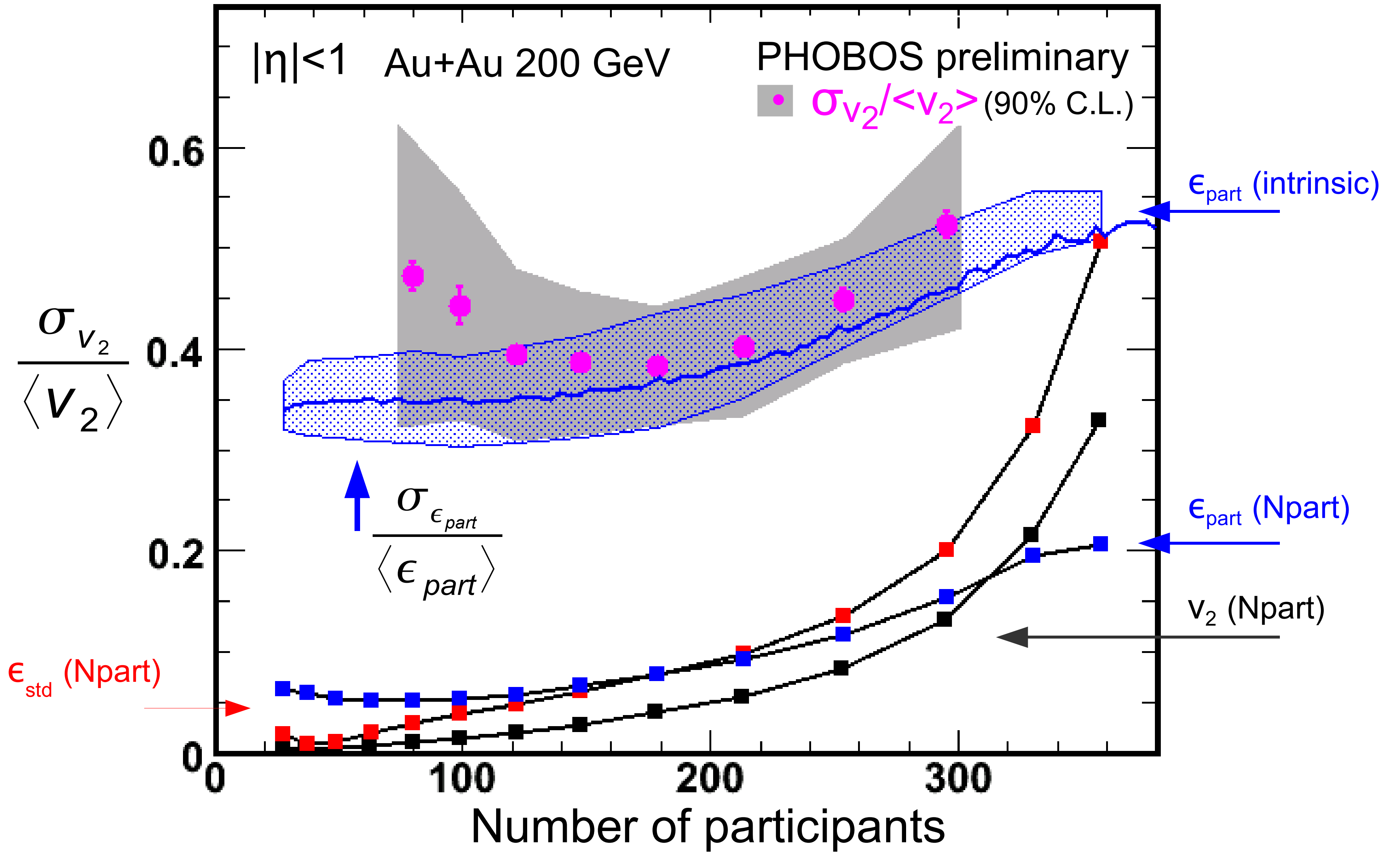
- PHOBOS has measured elliptic flow fluctuations in peripheral to semi-central Au+Au collisions at 200 GeV
  - Absolute fluctuations (σ_{v2}) are about 0.02
  - Relative fluctuations ($\sigma_{v2}/\langle v_2 \rangle$) are about 40%
  - The relative fluctuations are in striking agreement with predictions from the participant eccentricity
- Modeling of interaction points with MC Glauber interpreted event-by-event, the participant eccentricity model, appears to be able to explain both
  - The magnitude of the mean elliptic flow in Cu+Cu wrt Au+Au
  - The magnitude of the elliptic flow fluctuations in Au+Au

Backup slides

Systematic error sources



Contributions from Npart fluctuations



Fluctuations in Npart are calculated by folding f(Npart) with a Gaussian with mean and sigma as obtained from the centrality selection used in PHOBOS

Expected elliptic flow fluctuations