Fluctuations in Au+Au collisions at $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$

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Introduction

Fluctuations in various observables has been proposed as signatures of the Quark–Gluon–Plasma. Here, the measure ω_x, where x is either the charged particle multiplicity M_{ch} or summed deposited energy E, is explored in detail.

Andrew Jackson once proposed the following analogy why fluctuation and event–by–event analysis are of interest [1]:

> Stick a sheet of paper out of your window on a rainy day. Keeping it there for a long time — corresponding to averaging — the paper will become uniformly wet and one would conclude that rain is a uniform mist. If, however, one keeps the sheet of paper in the rain for a few seconds only, one observes the striking droplet structure of rain.

One of the motivations for studying fluctuations, is the abrupt change in the chiral order parameter $\langle\bar{\psi}\psi\rangle$ as predicted by Lattice QCD (see Figure). Such discontinuities in the order parameter are generally associated with large density and energy density fluctuations [2], which one can hope to observe in relativistic heavy–ion collisions.

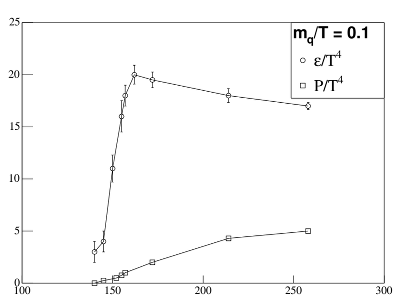


Figure 1: Lattice QCD calculations of energy density ε and pressure P as a function of temperature T, with only two flavours of light quarks (u and d quarks). The critical temperature T_c is here $\approx 150\,\mathrm{MeV}$. Adapted from [3].

The ω_x Fluctuation Measure

For a sample of N events, where for each event one measures N_i, $i = 1,\ldots,N$ times the variable x_{ij}, $j = 1,\ldots,N_i$, we can define the average and squared spread over all measurements over all events as

$$\bar{x} \equiv \frac{1}{\sum_{i=1}^{N} N_i}\sum_{i=1}^{N}\sum_{j=1}^{N_i} x_{ij} \quad s_x^2 \equiv \frac{1}{N_t - 1}\sum_{i=0}^{N}\sum_{j=1}^{N_i}\left(x_{ij} - \bar{x}\right)^2 \quad . \tag{1}$$

If the measurement x is made up of contributions from many particles, like for example total multiplicity M, total charged particle multiplicity M_{ch}, total charge C, net charge Q, or total transverse energy $E_\perp$, so that only one measurement of x is done in a single event, one can characterise the fluctuations in x by the ω measure

$$\omega_x = \frac{s_x^2}{\bar{x}} \tag{2}$$

If x is measured in various bins of some other observable(s) o, like say pseudo rapidity and polar angle (η,φ) and/or centrality, one can further define

$$\omega_{x,o_i o_j} = \frac{C_{x,o_i,o_j}}{\sqrt{\overline{x_{o_i}}\,\overline{x_{o_j}}}} \tag{3}$$

where

$$C_{x,o_i,o_j} = \frac{1}{N}\sum_i^N \left(x_{o_i} - \overline{x_{o_i}}\right)\left(x_{o_j} - \overline{x_{o_j}}\right)$$

is the ij^{th} entry of the covariance matrix C. For $o_i = o_j$ (3) becomes

$$\omega_{x,o_i} = \frac{s_{x_o}^2}{\overline{x_o}} \tag{4}$$

It is important to realise, that this measure is ***not*** an event–by–event measure, as it characterises the dispersion of the variable x over the full event sample. However, the dispersion *is* of course related to the underlying physical event fluctuations, that produce the observable x.

The ω_x measure was proposed by Baym, Blättel, Frankfurt, Heiselberg and Strikman, and they have made various calculations of ω_x, including various production mechanism and smearing effects [4]. The measure has been applied to experimental data by the WA98 collaboration [5–7] to M_{ch}, multiplicity of γ-like particles, and total $E_\perp$. The measure has also been applied to NA49 data on M_{ch} [8], and more recently by Jakobsen to BRAHMS $\sqrt{s_{NN}} = 130\,\mathrm{GeV}$ data on M_{ch} [9].

Characteristics of ω

If the observable x is distributed according to a Poissonian probability function, it is trivial to see that the measure ω_x becomes unity

$$\omega_x = \frac{\sigma^2}{\mu} = 1 \quad .$$

Hence, if the observable x is trivially distributed, one would expect ω_x exactly equal to 1.

For a binomial distribution with probability p, the fluctuation become

$$\omega_x = \frac{np(1-p)}{np} = 1 - p \quad , \tag{5}$$

where p is the probability of obtaining x successes out of n possible.

Consider N sources, each contributing with x_i (e.g., some number of particles) to the measurement x, so that

$$x = \sum_{i=1}^{N} x_i$$

If the number of sources and the x_i of each source are independent, so that

$$\bar{x} = \overline{N}\,\overline{x_i} \quad ,$$

and if the sources are independent, so that $\overline{x_i x_j} = \overline{x_i}\,\overline{x_j}, i \neq j$, we can write

$$\omega_x = \omega_{x_i} + \overline{x_i}\,\omega_N \tag{6}$$

where ω_{x_i} is the fluctuation of the contribution for each source x, and ω_N is the fluctuation in the number of sources, where the averages and spreads are taken over all sources and all events.

Fluctuations in Charged Particle Multiplicity — $\omega_{M_{\mathrm{ch}}}$

Heiselberg [1] notes that the mean number of charged particles in pp collisions can be parameterised as

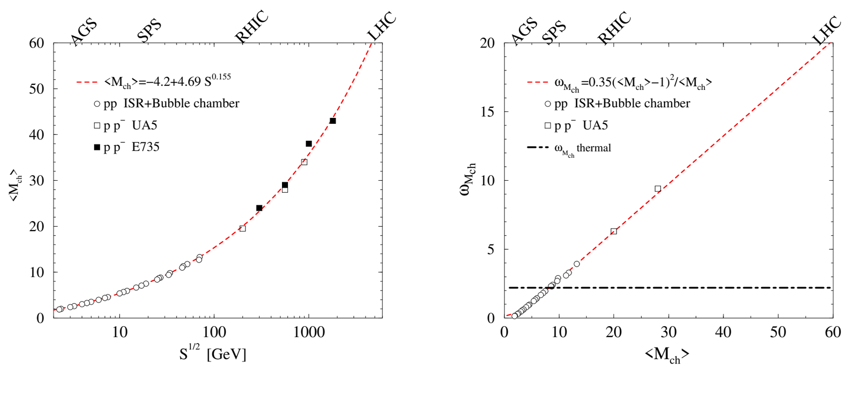
$$\langle M_{\mathrm{ch}}\rangle = -4.2 + 4.69\left(\sqrt{s_{NN}}\right)^{0.31} \quad , \tag{7}$$

integrated over all rapidities, as shown in Figure 2(a). KNO scaling in in high–energy ($\sqrt{s} > 2\,\mathrm{GeV}$) pp and $p\bar{p}$ collisions, means that multiplicity distributions scale with the average multiplicity. This in turn implies that all moments of the charged particle multiplicity distribution, and in particular the second moment, scale like

$$\langle M_{\mathrm{ch}}{}^q\rangle \propto \langle M_{\mathrm{ch}}\rangle^q \quad .$$

Thus the fluctuations in M_{ch} scale linearly with $\langle M_{\mathrm{ch}}\rangle$ as

$$\omega_{M_{\mathrm{ch}}} \approx 0.35\frac{\left(\langle M_{\mathrm{ch}}\rangle - 1\right)^2}{\langle M_{\mathrm{ch}}\rangle} \quad \text{for N+N collisions [1].} \tag{8}$$



(a) (b)

Figure 2: (a) $\langle M_{\mathrm{ch}}\rangle$ parameterisation. (b) KNO scaling of $\omega_{M_{\mathrm{ch}}}$. Adapted from [1].

This leads to a prediction of

$$\langle M_{\mathrm{ch}}\rangle = -4.2 + 4.69\cdot 200^{0.31} \approx 20 \tag{9}$$

$$\omega_{M_{\mathrm{ch}}} = 0.35\frac{(20-1)^2}{20} \approx 6.3 \tag{10}$$

for N+N collisions at $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$, the current RHIC energies.

Now, supposing that in a given collision, there are n_i N+N collisions, labelled by j, each producing $m_{i,j}$ charged particles, the sources are independent (uncorrelated), and the number of charged particles produced per N+N collision is independent of the number of N+N collisions

$$M_{\mathrm{ch}i} = \sum_{j=1}^{n_i} m_{i,j} \qquad \overline{m_i m_j} = \overline{m_i}\,\overline{m_j} \qquad \overline{M_{\mathrm{ch}}} = \overline{m}\,\overline{n} \quad ,$$

then we can write

$$\omega_{M_{\mathrm{ch}}} = \omega_m + \overline{m}\omega_n \quad , \tag{11}$$

where ω_m is the fluctuation in charged particles from the individual N+N collisions, and ω_n is the fluctuations in the number of N+N collisions.

Using (11) it is possible, to separate out the contribution to $\omega_{M_{\mathrm{ch}}}$ that stems from sampling over a finite range of impact parameters. The point is, that with decreasing b, more and more participant sources are available, but they need not be utilised; hence ω_n may be non–zero, but (11) allows separating out this finite impact parameter resolution.

Using an estimate of $\omega_m = 6.2$ as explained above, and the measured multiplicities at $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$ [10], one finds for the 5 % most central events

$$\omega_{M_{\mathrm{ch}}} = 6.2 + \frac{14630}{2\,346}1.1 \approx 13.6 \quad , \tag{12}$$

assuming full coverage in phase–space, for Au+Au collisions at $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$.

In the case that the particles are produced by a thermal source, where the mean number of particles in bosonic mode a is

$$\langle m_a\rangle = \frac{1}{e^{\frac{E_a}{T}} - 1} \quad ,$$

one finds [1] that the fluctuation in each state is given by

$$\omega_{m_a} = 1 + \langle m_a\rangle \quad , \tag{13}$$

so that the fluctuation in the total multiplicity $M = \sum_a m_a$ becomes

$$\omega_M = 1 + \frac{\sum_a \langle m_a\rangle^2}{\sum_a \langle m_a\rangle} \quad .$$

In [1] this is evaluated for a labelling momentum states of $\pi^{\pm}$ yielding

$$\omega_{M_\pi} = r\frac{1-r}{1+r} + (1+r)1.11 \approx 1.4 \quad ,$$

where r is the fraction of $\pi^{\pm}$ from resonance decays.

Assuming full coverage of phase space, the effect of finite impact parameter resolution

$$\omega_{M_{\mathrm{ch}}} \approx 8.7 \quad ,$$

for the 5 % most central events in Au+Au at $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$.

In the case of a phase transition to chiral symmetric and deconfined quark matter, the fluctuations are expected to many orders of magnitude higher than that from hadronic matter, whether the particle production is governed by KNO scaling or thermal constraints [1]. Heisenberg also notes, that the if the hadronisation of the quark matter is smooth it may very well wash out the fluctuation signal all together.

Fluctuations in Deposited Energy — ω_E

When a single detector element has an high occupancy the deposited energy in that detector will be a sum of the contributions from each particle impinging on the detector.

For the i^{th} event, assume that n_i sources each produce $m_{i,j}$ particles in the acceptance of a detector, or some bin in phase space, so that the total multiplicity and energy deposited in that detector becomes (i labels the event, while j labels the source and k labels the produced particle)

$$M_{\mathrm{ch}i} = \sum_j^{n_i} m_{i,j} \qquad E_i = \sum_k^{M_{\mathrm{ch}i}} e_k \quad , \tag{14}$$

where e_j is the energy deposited by the j^{th} particle. Following Section , one finds that

$$\overline{M_{\mathrm{ch}}} = \overline{n}\,\overline{m} \qquad \overline{E} = \overline{M_{\mathrm{ch}}}\,\overline{e} \quad .$$

Again following Section , the fluctuation in E can be rewritten as

$$\omega_E = \frac{s_E^2}{\overline{E}} = \omega_e + \frac{\overline{E}}{\overline{M_{\mathrm{ch}}}}\left(\omega_m + \frac{\overline{E}}{\overline{e}}\frac{\omega_n}{\overline{n}}\right) \tag{15}$$

Here, ω_e is the fluctuation in the energy deposited by a charged particle, ω_n is the fluctuation in the number of sources, $\bar{n}$ is the average number of sources, and ω_m is the fluctuation in the charged particle production at the sources. In this way, we can express the measured ω_E as a function of the measured $\overline{E}$ with the interesting ω_m as a parameter.

It is also worth noting, that ω_e in principle absorbs all detector inefficiencies and quirks, and fluctuations due to fluctuations in the energy loss of the charged particles impinging on the detector. However, using (15) it is in principle possible to factor out these contributions to the measured fluctuations in the summed energy deposited ω_E, as long as the assumption of $\overline{E} = \overline{M_{\mathrm{ch}}}\,\overline{e}$ and holds. That may *not* be the case, if the detector is such that it has a very long Landau tail in the energy spectrum, which will lead to a very large spread, and may hence completely dominate in (15).

Fluctuations Energy Deposited in the SMA

Event Sample

A fine segmentation of 25 even sized bins in the η range $[-2.5, 2.5]$ was used, and 4 bins in φ. In centrality c, 13 bins with variable bin sizes were used.

The fluctuations is calculated in separate η, φ matrices for each defined event class. The classification of events is done based on the centrality as determined from the TMA.

M_{ch} and E for a given η, φ, c bin is calculated as the acceptance corrected sum of the $N_{\eta,\varphi,c}$ contributions from detectors that are in that η, φ, c range.

$$E_{\eta,\varphi,c} \equiv \sum_i^{N_{\eta,\varphi,c}} E_i \frac{2\pi}{\Delta\varphi_i\Delta\eta_i}\sin\vartheta_i \tag{16}$$

More than 10 000 000 events within the η and centrality criteria was used. This gives a number of counts $> 1\,000\,000$ in each bin, satisfying the need for a large statistical sample to validate the assumptions of the central limit theorem.

Errors

The statistical error on ω_x is given by:

$$s^2_{\omega_x} = \frac{\omega_x^2}{N}\left(\frac{\omega_x}{\bar{x}} + 4\right) \quad . \tag{17}$$

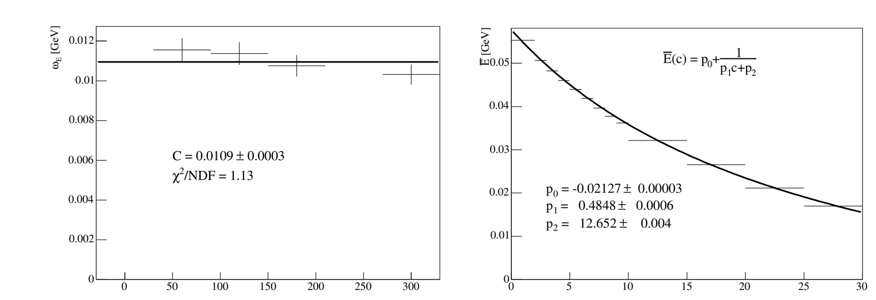
The systematic error, $s^2_{\omega_x'}$, is obtained from the systematic error on x by observing that for a constant a

$$\omega_{ax} = \frac{s^2_{ax}}{\overline{ax}} = \frac{\frac{1}{N-1}\sum_i\left(ax_i - \overline{ax}\right)^2}{\frac{1}{N}\sum_i ax_i} = \frac{a^2\frac{1}{N-1}\sum_i\left(x_i - \bar{x}\right)^2}{a\frac{1}{N}\sum_i x_i} = a\omega_x \quad .$$

Thus, the systematic errors on $\omega_{M_{\mathrm{ch}}}$ and ω_E are the same as the systematic errors on M_{ch} and E.

Polar angle φ and Centrality c Dependence

As can be seen on Figure 3(a), ω_E does not vary much over φ and hence the rest of the analysis is carried out summing over the full range in φ.



(a) (b)

Figure 3: (a) ω_E as a function of φ. (b) $\overline{E}$ as a function of c.

Figure 4 shows the measured fluctuations in deposited energy in the SMA.

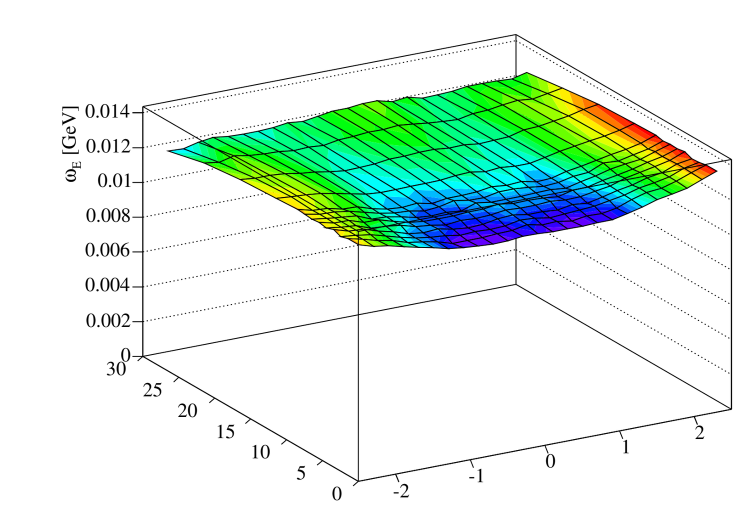


Figure 4: ω_E as a function of η and c.

A more striking feature of the analysis, shown in Figure , is that ω_E varies very little with c. This is an indication that very little changes in the region of $c \in [0\,\%, 30\,\%]$, which approximately covers from full to half overlap of the colliding nuclei.

ω_E as a function of $\overline{E}$

The equation

$$\omega_E = \omega_e + \frac{\overline{E}}{\overline{M_{\mathrm{ch}}}}\left(\omega_m + \frac{\overline{E}}{\overline{e}}\frac{\omega_n}{\overline{n}}\right) \tag{15}$$

expresses the intrinsic fluctuations ω_m in the particle production as a function of the measurable quantities $\overline{E}$, $\overline{M_{\mathrm{ch}}}$ and ω_E with the parameters: fluctuation in energy deposited per charged particle ω_e, average deposited energy per particle $\bar{e}$, fluctuations in number of sources ω_n and average number of sources $\bar{n}$. Since ω_E changes very little over $c \in [0\,\%, 30\,\%]$ and that $\overline{E} \propto \frac{1}{c}$ in that range (see Figure 3(b)), one can plot ω_E as a function of η and $\overline{E}$ rather than the measured c. This is shown in Figure 5.

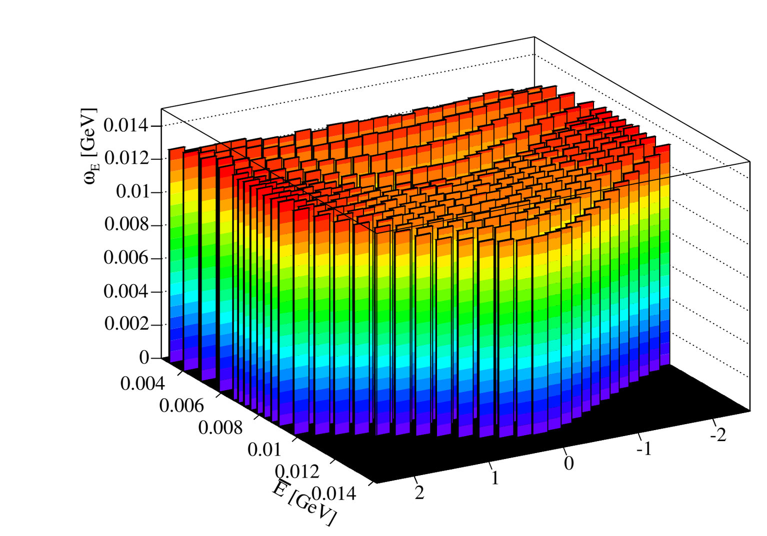


Figure 5: ω_E as a function of η and $\overline{E}$

To evaluate ω_m, the histogram in Figure 5 is sliced up along η, giving ω_E as a function of $\overline{E}$ for a given η. As $\overline{M_{\mathrm{ch}}} \propto \overline{E}$, we can change (15) to read

$$\omega_E = \omega_e + \overline{E}\frac{\omega_m}{\overline{M_{\mathrm{ch},0}} + \frac{1}{\overline{e}}\overline{E}} + \overline{E}\frac{\omega_n}{\overline{n}} \tag{18}$$

Each slice in η is fitted to the function

$$f(x) = p_0 + x\frac{p_1}{p_2 + xp_3} + xp_4 \tag{19}$$

The result of these fits, along with the χ^2 per degrees of freedom (NDF) are shown in Figure 6 summarises the average, spread and χ^2/NDF of $\overline{M_{\mathrm{ch}}}$, ω_m, and $\frac{\omega_n}{\overline{n}\overline{e}}$ from the fits.

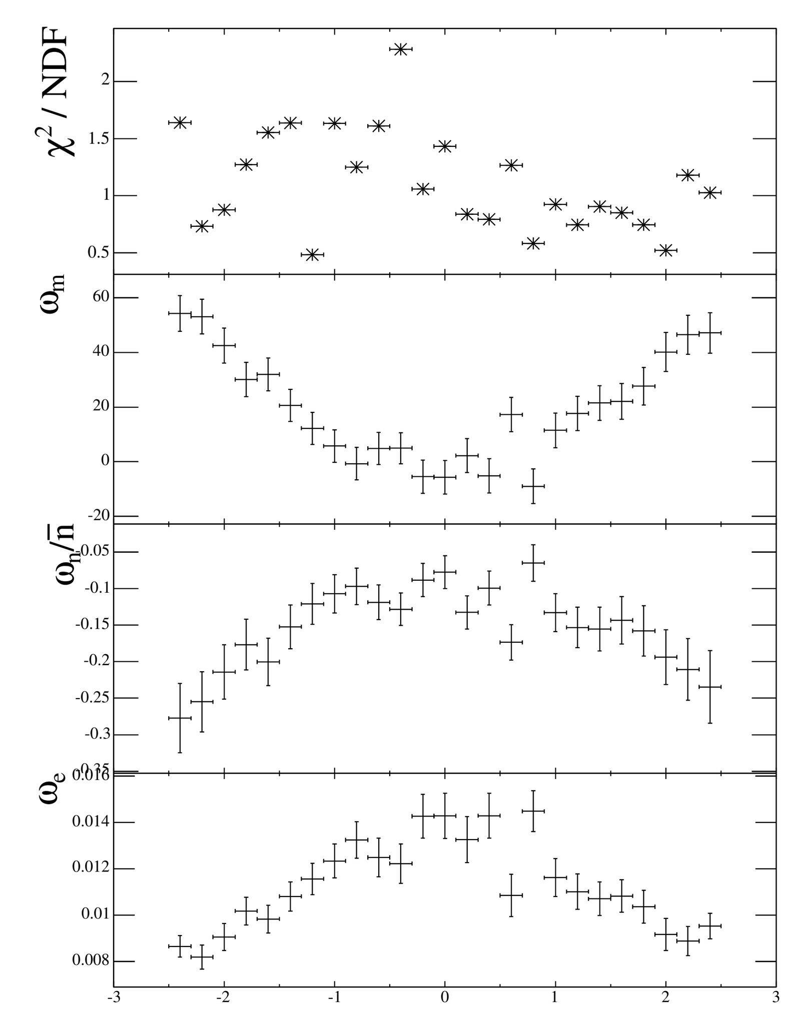


Figure 6: The χ^2/NDF and parameters of Figure 5 fitted in each η bin to (19). The average χ^2/NDF is ≈ 1.1.

Note, that the relatively large value of ω_e can be attributed to the integrating nature of the SMA. In general, a charged particle deposit energy according to a Landau distribution.

If, contrary to the assumption in Section , the number of particles produced is anti–correlated with number sources, then the cross term.

$$C_{n,m} = \frac{1}{N}\sum_k^N\left(n_k - \overline{n}\right)\left(m_k - \overline{m}\right) \quad ,$$

may become negative. This means that the fluctuations will drop as the number of source increases, or equivalently — the centrality falls. Then, the average of the total multiplicity would become

$$\overline{M_{\mathrm{ch}}} = \overline{m}\,\overline{n} + C_{mn} \quad ,$$

and an additional terms would be present in (15), all proportional to C_{mn} that is potentially negative. Hence, the assumption of independent particle production of the sources, does not seem to hold.

Comparison to HIJING Data

A BRAG detector simulation can in principle be used to evaluate any unidentifiable contributions to the fluctuations in particles produced per source ω_m, to obtain a bare ω_m' by

$$\omega_m' = \omega_m\frac{\omega_{m,S}}{\omega_{m,V}} \quad . \tag{20}$$

However, as stressed in that section, this interpretation of ω_m' is highly model dependent.

Figure 7 shows the result of applying (20) (where the ratio is that in obtained from simulations) to ω_m from Figure 6.

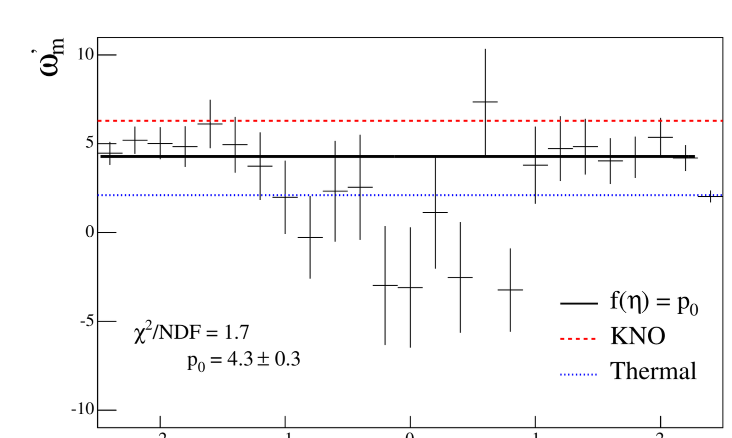


Figure 7: ω_m (from Figure 6) adjusted for secondary particle production, geometrical and detector acceptance, as well as other unidentifiable sources of fluctuations.

As can be seen from Figure 7 the bare fluctuations are, averaged over η, 4.3 ± 0.3 at with a rather poor χ^2/NDF of 1.7.

The main cause of errors in the figure, is the error on ω_m in Figure 6. The fact that it is not as smooth as one may have hoped shows up again in the bare ω_m, and is the main reason why the constant fit is relatively poor.

Assuming that the model calculations are sound, the value of η, 4.3 ± 0.3 is far above the Poissonian value of 1, and in between the value of 6.3 expected from KNO scaling at $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$, and the value of 1.4 from a thermal source (see also Figure 2(b)). Hence, the conclusion of the analysis is, that in terms of fluctuations, Au+Au collisions at relativistic energies behaves much like scaled N+N collisions. However, the level is not high enough to suggest that a transition to chiral symmetric and deconfined matter has been achieved, which would most probably result in fluctuations orders of magnitude higher [1].

References

[1] H. Heiselberg, *Phys. Rept.*, **351** (2001) 161–194, nucl-th/0003046.

[2] H. S. Walter Greiner, Ludwig Neise, *Thermodynamics and Statistical Mechanics*. Springer-Verlag, New York, 1997.

[3] T. Blum, L. Karkkainen, D. Toussaint, and S. Gottlieb, *Phys. Rev.*, **D51** (1995) 5153–5164, hep-lat/9410014.

[4] G. Baym, B. Blattel, L. L. Frankfurt, H. Heiselberg, and M. Strikman, *Phys. Rev.*, **C52** (1995) 1604–1617, nucl-th/9502038.

[5] WA98 Collaboration, M. M. Aggarwal *et al.*, nucl-ex/0108029.

[6] WA98 Collaboration, T. K. Nayak *et al.*, *Pramana*, **57** (2001) 285–300, nucl-ex/0103007.

[7] WA98 Collaboration, T. K. Nayak *et al.*, nucl-ex/0108026.

[8] G. Baym and H. Heiselberg, *Phys. Lett.*, **B469** (1999) 7–11, nucl-th/9905022.

[9] E. Jakobsen, Master's thesis, Niels Bohr Institute, December, 2001.

[10] BRAHMS Collaboration, I. G. Bearden *et al.*, *Phys. Rev. Lett.*, **88** (2002) 202301, nucl-ex/0112001.