Comparing Centrality Methods to Impact Parameter
BRAHMS Analysis Note # 37
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Abstract

Presented is comparative analysis of 5 different methods of centrality determination: 2 from the SMA and TMA multiplicities, 1 from the combined TMA and SMA multiplicities, and 2 based on the energy signals from TMA and SMA. The input data consists of 5000 Fritiof, 5000 Hijing, and 4000 UrQMd minimum bias events. The normal analysis chain was used to obtain the results.

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1 Introduction

In the everlasting effort to simplify our analysis code and methods, an investigation into the centrality determination compared to 3 event generators quoted impact parameter was carried out. 3 distinct methods (see section 2) was used, all based on the TMA and SMA data, in order to find the simplest methodology to that will give a good correlation with the event generator input.

2 The Methods

The 5 different centralities $c(M_{\mathrm{TMA}})$, $c(M_{\mathrm{SMA}})$, $c(E_{\mathrm{TMA}})$, $c(E_{\mathrm{SMA}})$, and $c(M_{\mathrm{TMA+SMA}})$ fall naturally into 3 groups[1]:

Single Detector Multiplicity (SDM) Based on the total multiplicity as obtained from *one* detector i.e., M_{TMA} or M_{SMA}.

Single Detector Energy (SDE) Based on the total energy signal from a *single* detector i.e., E_{TMA} or E_{SMA}.

Array Multiplicity (AD) Based on the scaled sum of the multiplicity of both detectors i.e., $M_{\mathrm{TMA+SMA}} = M_{\mathrm{TMA}} + C \cdot M_{\mathrm{SMA}}$.

2.1 Single Detector Multiplicity (SDM)

The SDM[2] centrality [1] is based on the a set of calibration parameters, describing polynomials in the primary vertex's z–cordinate v_z.

These polynomials are obtained by making a pass over minimum bias events, and for a number of v_z bins (typically 16), determined the multiplicity M_c value corresponding to the c% most central events. These (v_z, M_c) pairs are then plotted and fitted with a N^{th} degree polynomial, giving the cut functions $f_c(v_z)$ where c is the defined cut[3].

In the analysis, these function is evaluated at the current primary vertex position v'_z. The centrality is then the least of the c's so that the total multiplicity from the given detector exceeds the function value $f_c(v'_z)$.

Using this method, it is only possible to say, that a given event is as central or more, as the defined cuts (the cs).

It should be noted, that the total multiplicity M for the SMA and TMA[4] have

[1] In the following, I'll let $c(x)$ denote the centrality as obtained from quantity x, M_x denote multiplicity, and E_x. x_{TMA} is to be understood as x as obtained from the single detector TMA, and similar for x_{TMA}. $x_{\mathrm{TMA+SMA}}$ should be understood as the scaled sum of x_{TMA} and x_{SMA}.

[2] The BRAT modules `Br[Si|Tile]CentCalModule` implements the calibration, while `Br[Si|Tile]CentModule` implements the analysis routines described below.

[3] c is typically one of $0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.4, 0.5$ and so on.

[4] Determined by modules `BrSiRdoModule` and `BrTileRdoModule`.

been corrected for the pseudo–rapidity η coverage for a single event (see [2]), so that the $\frac{d^2N}{dv_z d_M}$ distribution is more or less flat. Therefore, f_c are usually 1^{st} order polynomials[5].

2.2 Single Detector Energy (SDE)

The SDE[6] centrality is obtained very much like for SDM (see section 2.1 above.).

In the calibration, however, rather than finding the multiplicity M_c corresponding to each cut in each v_z bin, we find the energy E_c corresponding to the $c\%$ most central events. Again, the pairs (v_z, E_c) are plotted and fitted with a N^{th} degree polynomial, giving the cut functions $g_c(v_z)$.

Again, in the analysis, the functions $g_c(v'_z)$ are evaluated, and the centrality is then the least of the c's so that the total energy signal from the given detector exceeds the function value $g_c(v'_z)$.

As above, it is only possible to say, that a given event is as central or more, as the defined cuts (the cs).

By trial and error, the best[7] degree of the polynomials was found to be 4. The result of the calibration is shown in figure 1.

The advantage of the SDE centrality over the SDM centrality, is that it is not sensitive to the various calibrations that goes into converting the ADC signals to multiplicities[8] — only the pedestal, pulser, and gain calibrations affect the SDE centrality calibration. One may argue that this ignores the fact that much of the energy deposited in the detector elements in fact stems from (slow?) secondary particles, and that these should be corrected for. This will be explored more in section 3 below.

2.3 Array Multiplicity (AM)

The AM centrality also depends on a set of calibration parameters[9]. These parameters define to piece-wise function:

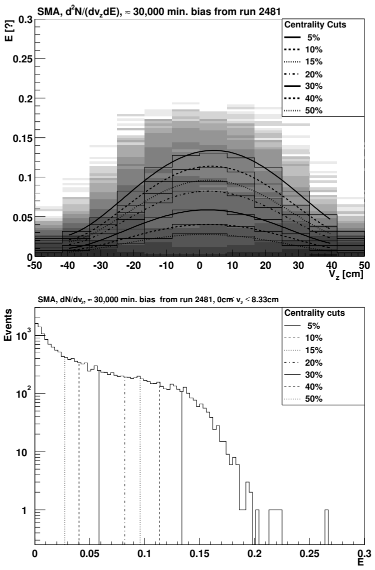
[5] Note, that the η correction is a 9^{th} degree polynomial in v_z for the SMA and a 4^{t} degree polynomial in v_z for the TMA. See also the BRAT modules `BrSiRdoModule` and `BrTileRdoModule`, as well as calibration classes `BrSiTmpCalibration` and `BrTileTmpCalibration`.

[6] The implementation of the below described algorithms are not (yet?) present in BRAT. However, the corresponding modules can be found in my CVS area in classes `BrMultCentCalModule` implements the calibration, while `NewCentModule`.

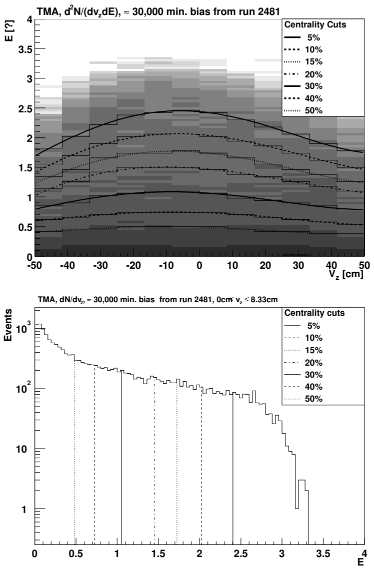
[7] Gave best χ^2 and gave consistent results.

[8] To get the single element multiplicity a correction function is applied in both the SMA and TMA. For the TMA, this is a 6^{th} degree polynomial in η and ring position. For the SMA, it's a 8^{th} degree polynomial in η. See also [2] and the BRAT modules `BrSiRdoModule` and `BrTileRdoModule`, as well as calibration classes `BrSiTmpCalibration` and `BrTileTmpCalibration`.

[9] See also `BrMultCentModule` and calibration in `BrMultCentTmpCalibration`.



(a) SMA



(b) TMA

Figure 1: Upper panel shows the cut functions. Lower panels show the a single vertex bin.

$$h(M_{\mathrm{TMA+SMA}}) = \begin{cases} 9^{\mathrm{th}} \text{ polynomial } M_{\mathrm{TMA+SMA}} > 100 \\ 7^{\mathrm{th}} \text{ polynomial } M_{\mathrm{TMA+SMA}} \leq 100 \end{cases} \tag{1}$$

How this parameters was obtained is unknown to me[10].

In the analysis, the function $h(M_{\mathrm{TMA+SMA}})$ is evaluated to directly give the centrality. Hence the AM centrality is continuous (where as the SDM and SDE centralities are discreet).

However, from the general error propagation, one finds that

$$\Delta M_{\mathrm{TMA+SMA}} = \Delta M_{\mathrm{TMA}} + C \cdot \Delta M_{\mathrm{SMA}} \quad C = 1.5476,$$

and so relatively[11] more weight is put on the SMA signal than on the TMA signal.

3 The Role of Secondaries

If, as there seems to be good reason to believe, that the number of secondary particles scales with the number of primary particles produced in the collision, then it is irrelevant to the centrality determination whether one corrects for the number of secondaries. The secondaries will simply make the dN/dM curve wider, but the top $x\%$ is still the top $x\%$ of the distribution. If there's a constant background of secondaries, it will move the left end–point of dN/dM away from 0, but will not change the integral of the distribution.

Being slightly more formal. Suppose that the number of secondaries M_{s} is given as

$$M_{\mathrm{s}} = a \cdot M_{\mathrm{p}} \quad ,$$

where M_{p} is the number of primaries, so that the measured number of particles is

$$M = M_{\mathrm{s}} + M_{\mathrm{p}} = (1+a)M_{\mathrm{p}}$$

then, if we define the cut multiplicity M_c corresponding to the centrality c by

$$c = \frac{\int_{M_c}^{M_{\mathrm{max}}} dM \, \frac{dN}{dM}}{\int_0^{M_{\mathrm{max}}} dM \, \frac{dN}{dM}}$$

Writing

$$\begin{aligned} M_c &= (1+a)M_{\mathrm{p,c}} \\ M_{\mathrm{max}} &= (1+a)M_{\mathrm{p,max}} \end{aligned}$$

and changing integration variable

$$M = (1+a)M_{\mathrm{p}} \quad \Rightarrow \quad dM = (1+a)dM_{\mathrm{p}} \quad ,$$

[10] Steve and Hiro should provide us with the code.

[11] The value of C is is taken from `BrMultCentModule`.

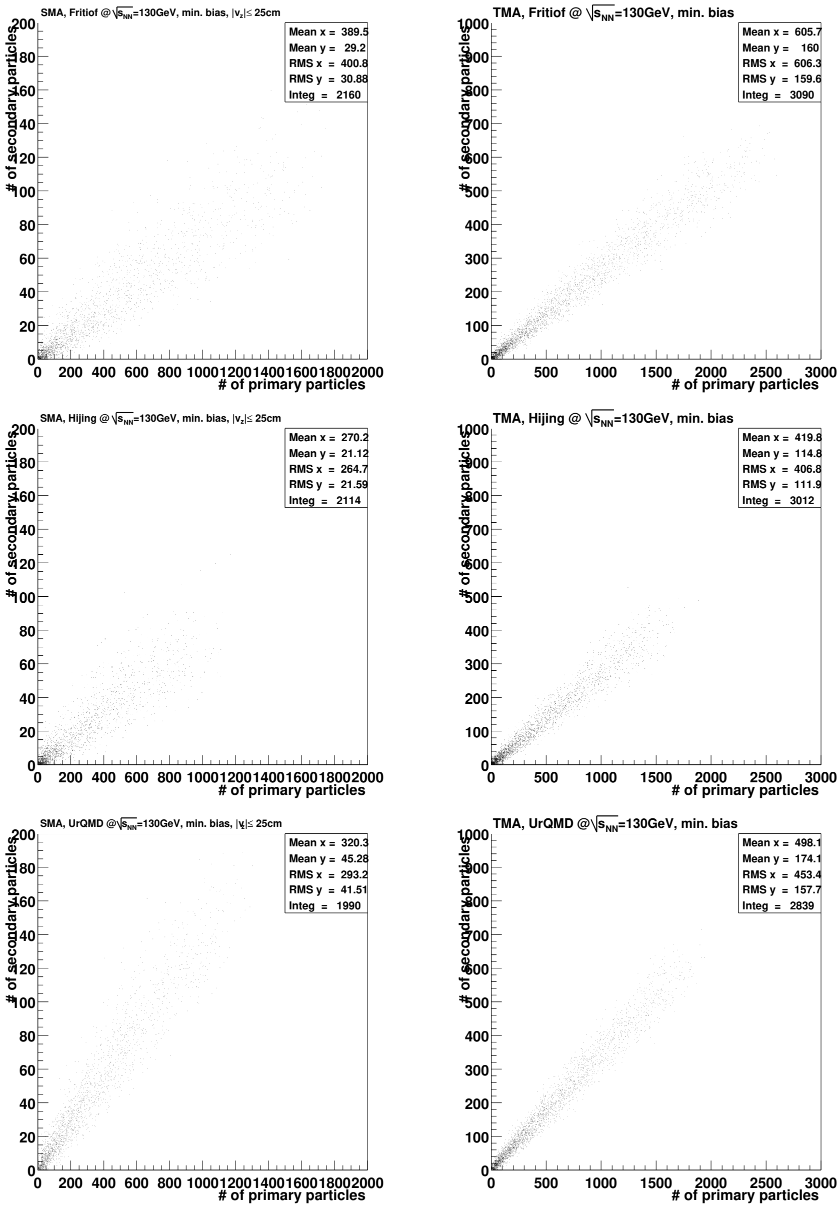
we find

$$c = \frac{\int_{M_{\mathrm{p,c}}}^{M_{\mathrm{p,max}}} (1+a) dM_{\mathrm{p}} \frac{dN}{(1+a)dM_{\mathrm{p}}}}{\int_{0}^{M_{\mathrm{p,max}}} (1+a) dM_{\mathrm{p}} \frac{dN}{(1+a)dM_{\mathrm{p}}}} = \frac{\int_{M_{\mathrm{p,c}}}^{M_{\mathrm{p,max}}} dM_{\mathrm{p}} \frac{dN}{dM_{\mathrm{p}}}}{\int_{0}^{M_{\mathrm{p,max}}} dM_{\mathrm{p}} \frac{dN}{dM_{\mathrm{p}}}} \quad ,$$

so we see that one can equally well cut in the measured multiplicity than the multiplicity corrected for secondaries.

The assumption that the number of secondaries scales with the number of primaries are justified by the analysis of simulated data, using Fritiof, UrQMD and Hijing. Figure 2 shows the correlation of primary particles with the number of secondaries. Figure 3 show the correlation between the energy deposited by primaries with the same of primaries.

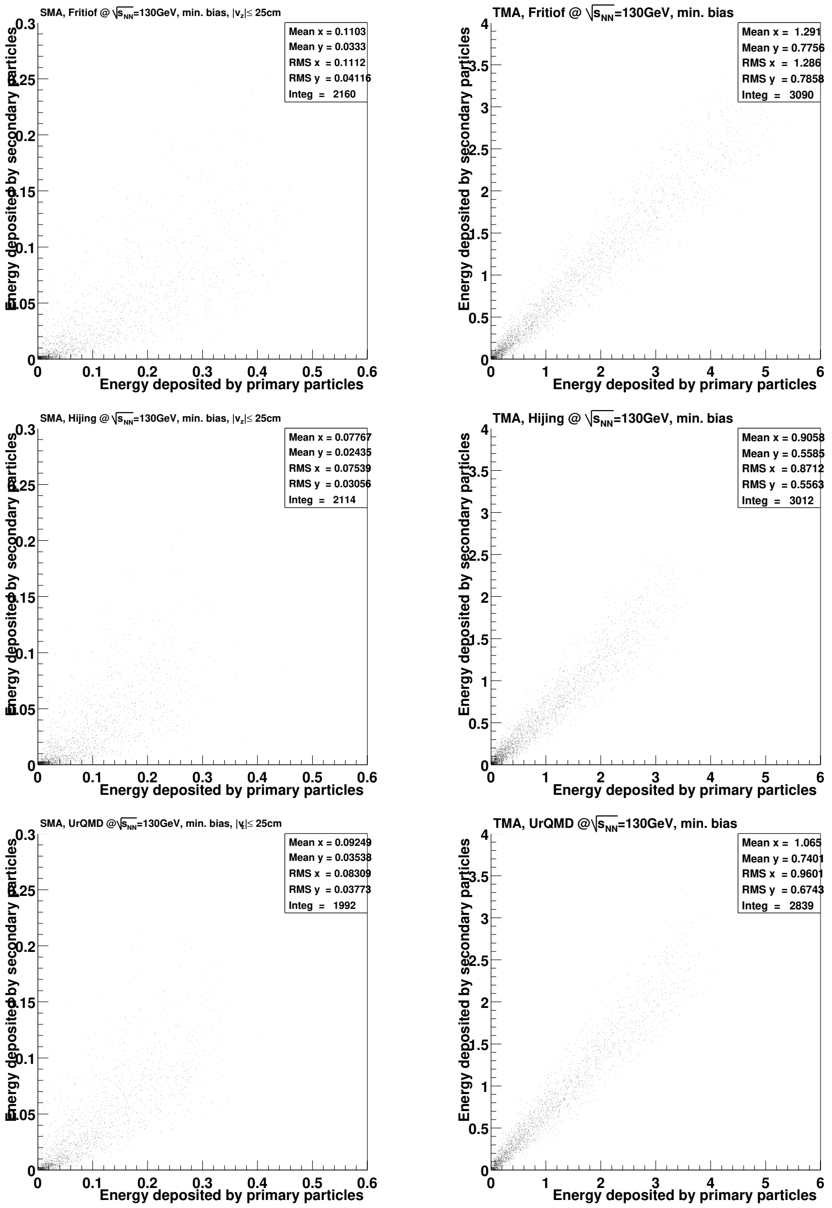
As a side effect of this analysis, I also obtained a mean value of the deposited energy per particle, for each of the three event generators. As been postulated many times, but I've never actually seen the plots, I find that to a large extent the mean energy deposited per particle, is independent of the event generator. The plots are reproduced for you're reference in appendix B.



(a) SMA

(b) TMA

Figure 2: Correlation of # of primaries to # of secondaries.



(a) SMA (b) TMA

Figure 3: Correlation of energy deposited by primaries with energy deposited by secondaries.

4 Comparing Centrality to Impact Parameter

An analysis of 5000 Fritiof, 5000 Hijing, and 4000 UrQMD minimum bias events[12] at $\sqrt{s_{NN}} = 130\text{GeV}$ was carried out, using the standard tools as provided by BRAT and BRAG.

A skeleton KUMAC for the BRAG job–control is reproduced in appendix A.

After passing the event generator data through BRAG, digitisation of the GEANT hit structures was performed, using the standard BRAT digitisation modules for BB, TMA and SMA[13 14]. It should be noted that I defined some user modules[15], so that I could carry over a summary of the original hits in the TMA and SMA[16], but this does not effect the subsequent analysis in any way.

Using the output of the digitisation pass above, I analyse these events, just as I would analyse raw data[17] i.e., I use the standard BRAT RDO modules to obtain the multiplicities and energy signals, as well as the standard centrality modules[18]. In addition, I also determined a SDE centrality for both SMA and TMA. A primary vertex cut of $|v_z| < 50\text{cm}$ was applied, since all calibrations seem valid in that interval.

The SDM and SDM centrality calibrations used in this analysis is based on $\approx$ 300,000 minimum bias events from run 2481 of last year.

The result of the analysis outlined above is shown in figures 4 to 6.

The seemingly large range over impact parameter that is assigned a low (0–5%) centrality in the case of the Fritiof event generator, simple comes about by the fact that Fritiof has a higher multiplicity than was seen in the data last year. Therefore, some events will have a higher multiplicity/energy signal than what was deemed the maximum multiplicity[19]/energy deposited in the calibration, and hence will be assigned a low centrality. Hence, this seemingly odd shape of the curves near low impact parameter is not to be alarmed about.

The centralities determined by the SDE method corresponds well to the numbers given in table 6 of [3], which is reproduced in part below in table 1.

Figure 7 shows the correlation between the two detector systems. Figure 8 shows the correlation of the SMA and TMA SDE centrality compared to the SMA+TMA AM centrality.

[12] The event generator files are not directly available to everyone, but I'll happily make *all* files available should the need arise.

[13] Note, that the actual calibrations was used to smear the data, rather than the parameters from the ASCII parameter file, as this gave far more consistent results.

[14] I'd very much have liked to get the ZDC data too, but unfortunately, no digitisation module exists at this time.

[15] These are available from my CVS area as `SMASimulModule` and `TMASimulModule`.

[16] I store the summaries in objects of class `MultSimul`, also available from my CVS area.

[17] As done by using `BrGlbPackage`, though I took out the ZDC module, since no such data is available in the input.

[18] Corresponding to the SDM and AM centrality methods outlined above.

[19] In the AM module `BrCentModule`, there's a sharp hard coded cut–off at 1680. Events with

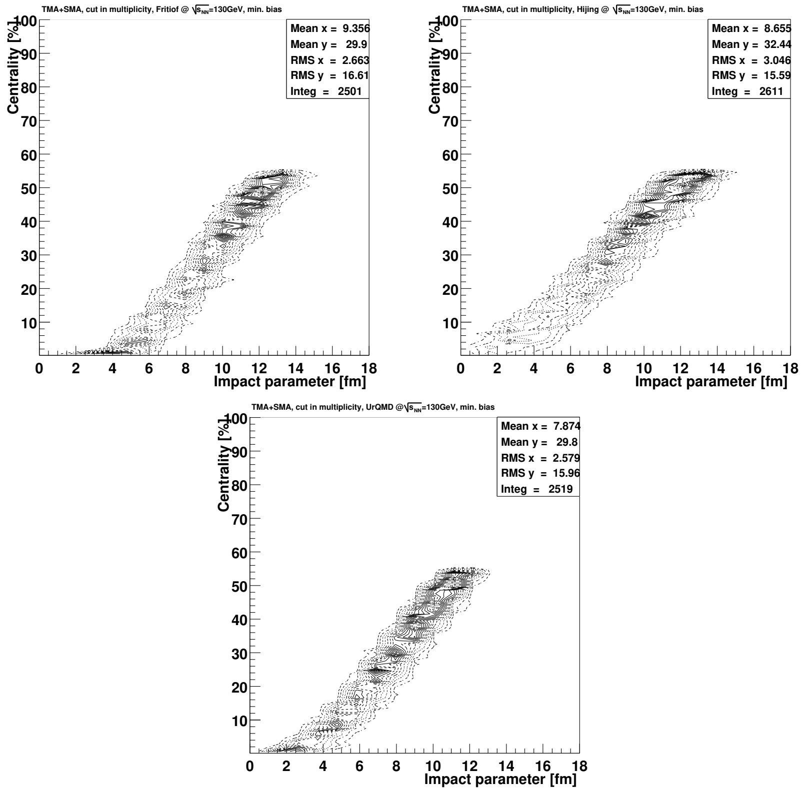
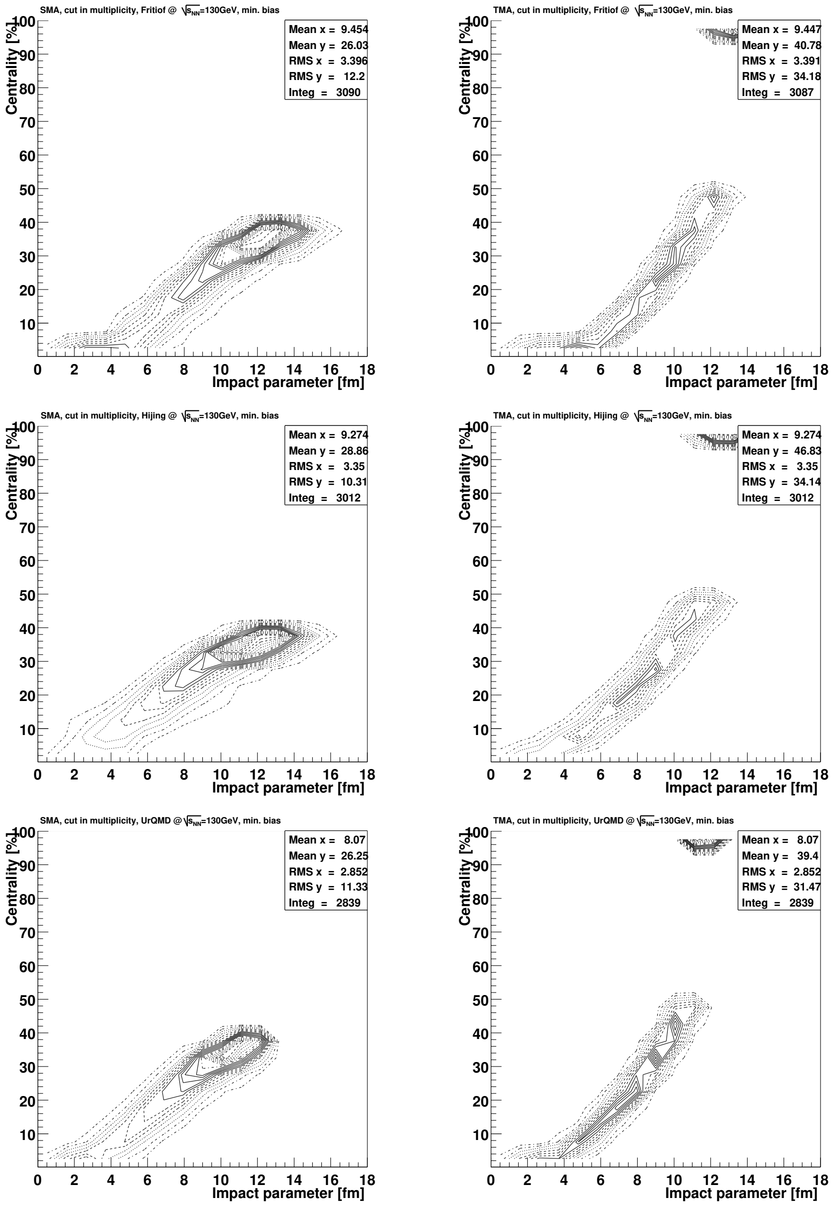


Figure 4: Impact parameter versus AM centrality.

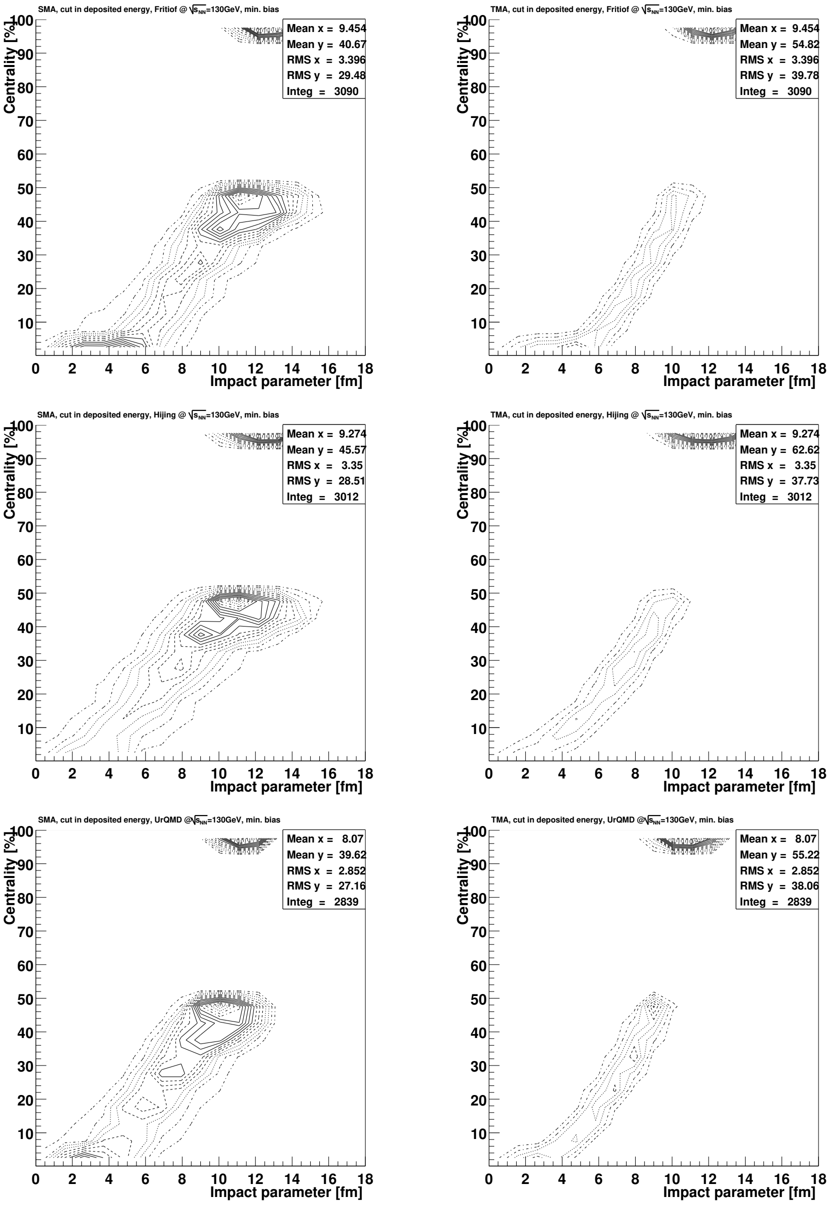
Centrality [%]	**Approximate** b **range** [fm]
0 − 10	0 − 5
10 − 20	5 − 7
20 − 30	7 − 8.5
30 − 40	8.5 − 10
40 − 50	10 − 11

Table 1: Centrality cuts with corresponding impact parameter range (From Hijing)



(a) SMA (b) TMA

Figure 5: Impact parameter versus SDM centrality.



(a) SMA (b) TMA

Figure 6: Impact parameter versus SDE centrality.

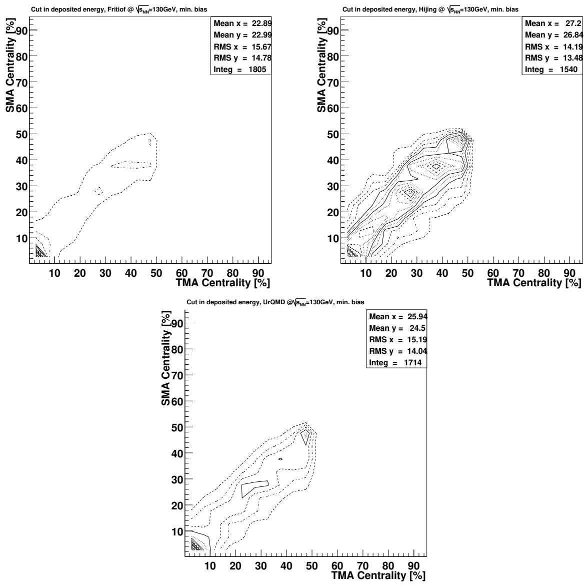
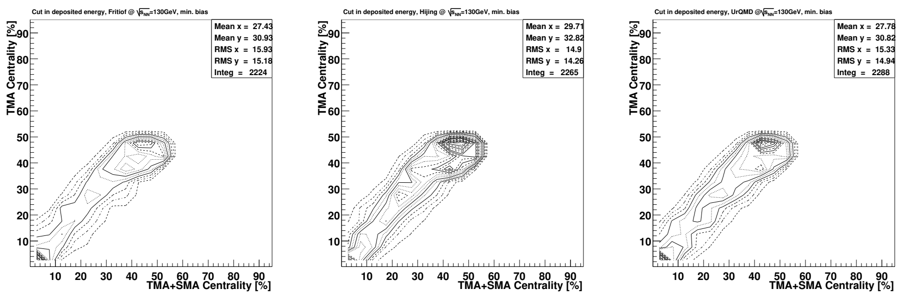
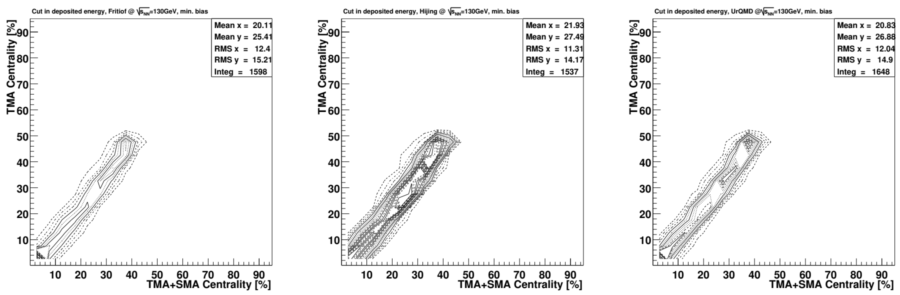


Figure 7: Correlation of centralities determined by SMA and TMA, using a SDE method.

a higher multiplicity than 1680, are always assigned centrality 0.



(a) SMA



(b) TMA

Figure 8: Centralities determined by SMA and TMA, using a SDE method, compared to the SMA+TMA AM centrality.

5 Conclusions

In my mind, it is evident, when it comes to determining the centrality of a collision, that there's little advantage in cutting in the multiplicity measured by the TMA or SMA, or the combined measurement, over making cut's in the measured deposited energy.

There's a nice correlation between the determined centrality and the event generators quoted impact parameter. Also, the SDE methods gives consistent results in both the SMA and TMA, though the SMA seems to perform a bit worse than the TMA. This in itself should make the AM method unattractive in it self.

There's a loads of other reason why I believe we should give up the AM centrality method in favour of the SDE centrality method. First and foremost, the SDE method is simpler and does not call for many corrections and parameterisations, as the SDM and AM methods does. Secondly, it is much faster to do the centrality calibrations using the SDE method, since one does not need to have the full multiplicity calibrations done to do the centrality calibration.

This last property of SDE should in itself be attractive for people wishing to do $\frac{dN}{d\eta}$, $\frac{d^2N}{m_\perp dm_\perp dy}$, $\frac{\bar{p}}{p}$ and other analysis using the two spectrometers, since the centrality calibrations can be done after the successful store of $\approx$ 1,000,000 minimum bias events[20]. This, adopting the SDE centrality, the BRAHMS collaboration should much faster be able to present centrality dependent results.

References

[1] C. E. O. Jørgensen and C. H. Christensen, "Centrality estimates using the scintilator tiles in the multiplicty array," BRAHMS Analysis Note 22, Niels Bohr Institute, University of Copenhagen, Febuary, 2001.

[2] H. Ito and S. Sanders, "$dn/d\eta$ Analysis by Silicon and Tile," BRAHMS Analysis Note 26, New York University, Texas A&M University, February, 2001.

[3] C. E. O. Jørgensen, "$\frac{dN}{d\eta}$ at Au+Au at $\sqrt{s_{NN}} = 130$AGeV," Master's thesis, University of Copenhagen, 4, 2001.

[20] As far as I remember, of the top of my head, this is no more than an afternoon of running.

A BRAG KUMAC

Below, the following shell variables are used:

Variable	Description
`$func`	File containing Fortran77 analysis subroutines
`$format`	Input file format (e.g., 7 for OSC1997a)
`$input`	Input data file
`$output`	Output file
`$nevents`	Number of events in input file

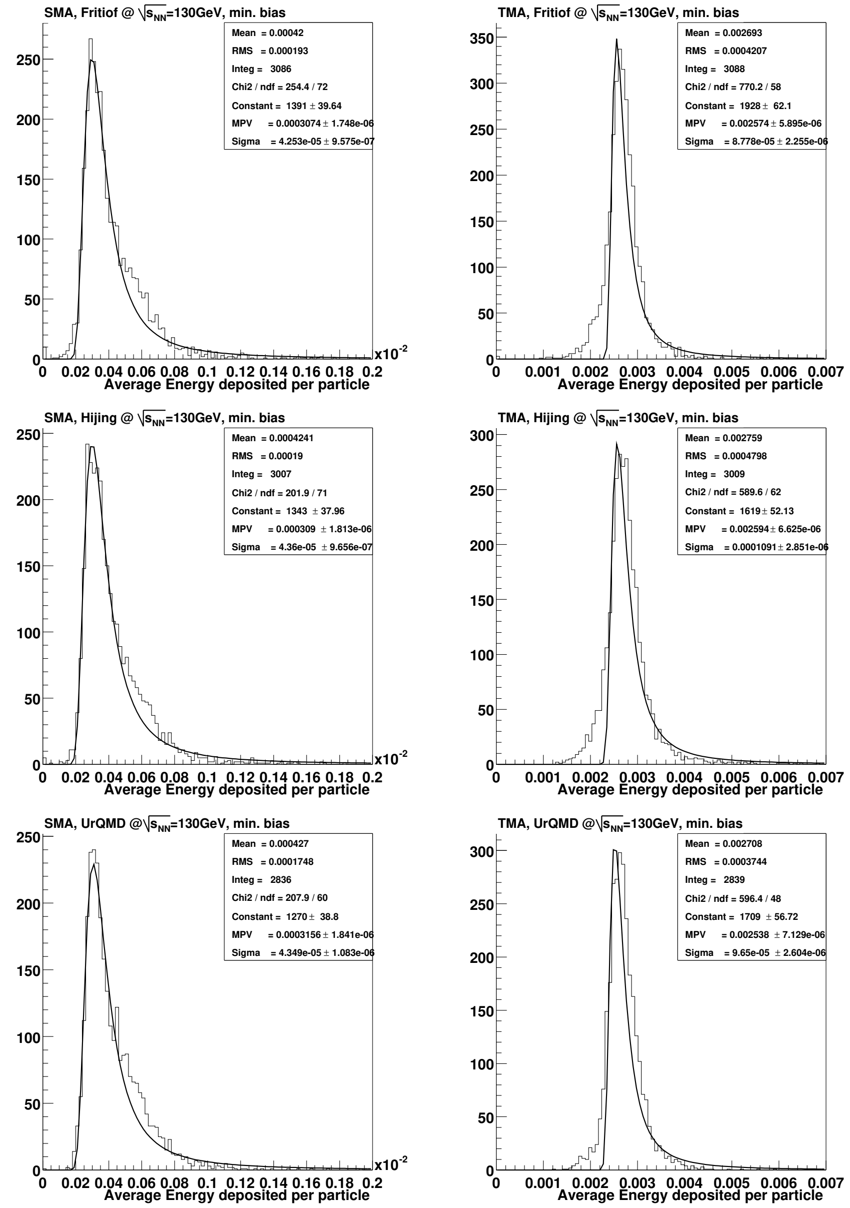
Table 2: BRAG KUMAC shell variables

```
MACRO SETUP
    ** Debugging setup
    KUIP/MESSAGE '=> Debugging setup'
    * GEANT/CONTROL/SWITCH 3 1
    GEANT/CONTROL/DEBUG  off

    ** Geometry
    KUIP/MESSAGE '=> Start setting the geometry'
    SETUP/GEOINI

    ** Shut of most stuff
    KUIP/MESSAGE '=> Turning MIDS,FMS1, and FMS2 volumes off'
    SETUP/SET_TREE mids z
    SETUP/SET_TREE fms1 z
    SETUP/SET_TREE fms2 z
    SETUP/SET_TREE zdc  z
    SETUP/SET_TREE zdc  z
    SETUP/SET_TREE shld z
    SETUP/SET_TREE flor z
    SETUP/SET_TREE dx   z

    ** Turn on all global stuff
    KUIP/MESSAGE '=> Turning BB,ZDC,MULT,BEAM,SHLD,FLOR and DX volumes on'
    SETUP/SET_TREE bb   osa
    SETUP/SET_TREE mult osa
    SETUP/SET_TREE beam osa
    * SETUP/SET_TREE zdc  osca
    * SETUP/SET_TREE shld osa
    * SETUP/SET_TREE flor osa
    * SETUP/SET_TREE dx   osa
```

```
** Finalise the setup
KUIP/MESSAGE '=> Finishing the geometry'
SETUP/GEODEF
SETUP/GEOFIN

** For debugging
* KUIP/MESSAGE '=> Draw it'
* GEANT/GRAPHICS_CONTROL/SATT mult seen 0
* GEANT/GRAPHICS_CONTROL/SATT bevi seen 0
* GEANT/GRAPHICS_CONTROL/DOPT hide on
* GEANT/GRAPHICS_CONTROL/NEXT
* GEANT/DRAWING/DRAW cave 45. 45. 0. 10. 10. .01 .01

** Read the analysis functions
KUIP/MESSAGE '=> Call analysis routine'
FORTRAN/CALL $func

** Setup the detector
KUIP/MESSAGE '=> Initialise detector'
FORTRAN/CALL init_detector

** Particles per trigger
KUIP/MESSAGE '=> 1 particle per trigger'
USER/CONTROL/EVSPLIT 1

** Do not store space-time points
KUIP/MESSAGE '=> Not storing space time points (card SPAC)'
USER/CONTROL/CARDS spac 1

** Set input and input format
KUIP/MESSAGE '=> Setting the input file to $input of format $format'
USER/CONTROL/CARDS ukin $format $input

** Set kinematically allowed region
KUIP/MESSAGE '=> Setting the kinematic region'
GEANT/CONTROL/KINE 1 0 100 0. 180. 0 360 0 49

** Set vertex distribution
**                <x> varX  <y> varY  <z>   varZ   maxX  maxY maxZ
KUIP/MESSAGE '=> Setting the vertex'
USER/CONTROL/SPOT 0.  1.    0.  1.    -0.95 77.16  2.    2.   50.

** Set the output file
KUIP/MESSAGE '=> Setting the output file to $output'
FORTRAN/CALL gbrfile('$output')
```

```
    ** Loop over rotations
    * KUIP/MESSAGE '=> Loop over rotations'
    * DO i = 0, $nrot
    * USER/CONTROL/ROTANG [i]*$angle

    ** Do the actual event analysis
    KUIP/MESSAGE '==> Analysing $nevent events'
    USER/CONTROL/ANALYZE gbrana $nevent 10 0
    *ENDDO

    ** Close the output file
    FORTRAN/CALL gbrend
RETURN
```

B Mean Energy Deposited per Particle



(a) SMA

(b) TMA

Figure 9: Mean energy deposited per particle (both primaries and secondaries). The distributions are fitted to a Landau distribution, and shows reasonably good agreement within the errors.